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The Theory of Hadronic Collisions

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The Theory of Hadronic Collisions

- Hadron Classification leads to development of The Quark Model.
- The study of Hadron Structure leads to The Parton Model.
- Understanding how to use Non-Abelian Gauge Theory, combined with the insights of the Quark and Parton Models leads to the development of « QCD » the fundamental theory of the strong interactions.

Hadron Classification: The Hadron Zoo

Starting in the mid-1940's, many hadronic states, both mesons and baryons, were discovered. Many were long-lived, decaying through the weak interaction, implying that they were not mere excitations of known particles.

Some could be grouped into isospin multiplets but the "strange" particles (K, Λ, Σ) did not fit in as expected.

M. Gell-Mann

Hadron Classification

Progress in classifying the hadrons came with Gell-Mann's introduction of Strangeness (S) as an additive quantum number. This explained some puzzles about the strange mesons and baryons (now known as K's and Σ's) and allowed them to be grouped into Isospin multiplets (K^+, K^0), (K^0, K^-), $(\Sigma^-, \Sigma^0, \Sigma^+)$. It also led to the Gell-Mann - Nishijima Relation

$$Q = I_3 + \frac{Y}{2} = I_3 + \frac{B+S}{2}$$

which connects isospin and strangeness violation with charge conservation.

M. Gell-Mann

SU(3) and the Eightfold Way

More mesons and baryons were discovered and were fit into a bewildering array of isospin multiplets. Gell-Mann and Ne'eman (independently) combined SU(2) isospin with strangeness to form an SU(3) symmetry.

The pseudoscalar and vector mesons as well as the spin $\frac{1}{2}$ baryons fit into octet representations of SU(3) which contains an isotriplet, a singlet and two isodoublets. The idea that the octet of SU(3) was the basic unit of organization was called the Eightfold Way.

The Eightfold Way

The meson and baryon octets:

	K^0		K^+			K^{*0}		K^{*+}			n		p
π^-		π^0,η		π^+	ρ^-		ρ^0,ω		ρ^+	Σ^-		Σ^0,Λ	Σ^+
	K^-		$\overline{K}^0$			K^{*-}		$\overline{K}^{*0}$			Ξ^-		Ξ^0

The eightfold way held that representations were octets or combinations of octets:

8x8 = 1 + 8 + 8 + 10 + 10* + 27

Δ^-		Δ^0		Δ^+		Δ^{++}
	Σ^{*-}		Σ^{*0}		Σ^{*+}	
		Ξ^{*-}		Ξ^{*0}		

The known spin 3/2 baryons were not in an octet, so were they in a 10 or a 27?

The Eightfold Way

Gell-Mann predicted that the multiplet was a decuplet. The missing resonance was the S=-3 Ω^-. He even predicted its mass and that it would decay via the weak interaction.

Δ^- Δ^0 Δ^+ Δ^{++}

Σ^{*-} Σ^{*0} Σ^{*+}

Ξ^{*-} Ξ^{*0}

Ω^-

Two years later, Samios and collaborators at Brookhaven found the Ω^- just as predicted. This was the great triumph of SU(3).


M. Gell-Mann

The Quark Model

Gell-Mann and Zweig each introduced the notion of hadron constituents lying in the fundamental representation of SU(3). Zweig called them "Aces", and Gell-Mann called them "Quarks". They necessarily had fractional charges.

Zweig called for a search to be made for the Aces, but Gell-Mann considered his Quarks to be mere book-keeping devices, with no intrinsic physical meaning.

The Introduction of Color

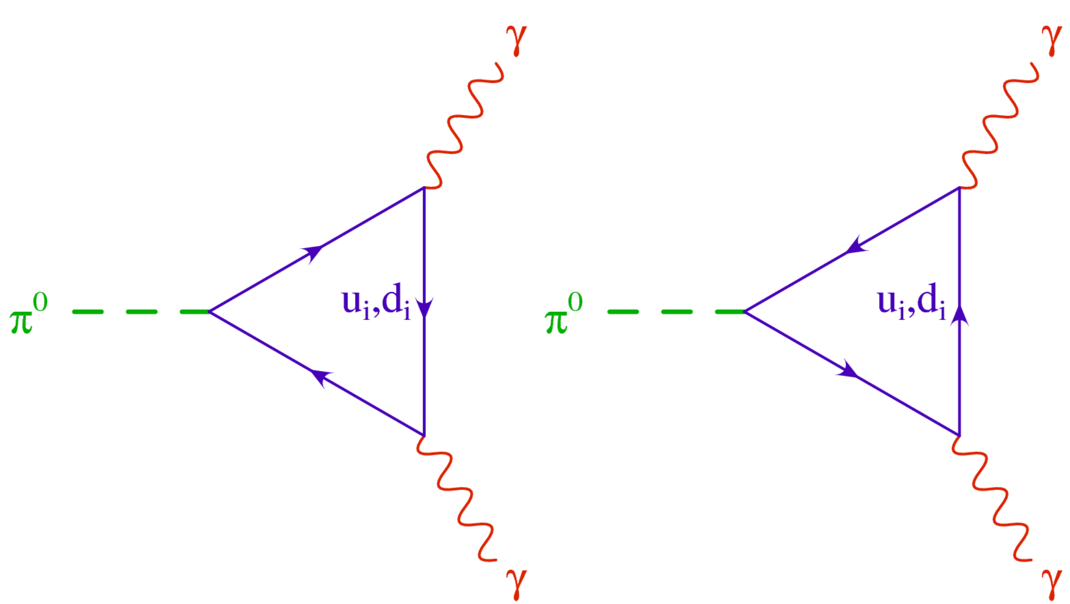
One expects the baryons to have ground state wave functions, which would be symmetric. The Isospin assignment implied that part of the wave function was also symmetric.

But baryons are fermions, so where is the antisymmetry? Greenberg and Han and Nambu each introduced a three-valued internal quantum number (or parastatistics) that would provide the needed antisymmetry.

There was still no model of dynamics and gluons (if mentioned) they were taken to be colorless.

Colored Quarks

The notion of colored quarks (or that quarks obey para-fermi statistics of rank three) was supported by the understanding of the role of the anomaly in π^0 decay.



If the quarks are uncolored, the rate would be off by a factor of three. The color sum gets the right result.

Summary: The Quark Model

The problem of hadron classification led to the theories of

- strangeness
- SU(3) and the Eightfold Way
- SU(3) and the Quark Model

Exploring the consequences of the quark model led to the notions of gluons and of a three-valued quantum number which came to be called color.

The Structure of the Proton: Elastic e-p Scattering

Elastic Scattering of an electron off of a static spin ½ point charge (Mott Scattering) is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} = \frac{4\alpha^2 E^2}{Q^4}\cos^2(\theta/2), \quad Q^2 = 4E^2 \sin^2(\theta/2)$$

If the static charge is replaced by a point-like Dirac particle of mass M, the formula is:

$$\frac{d\sigma}{d\Omega} = \left[\frac{d\sigma}{d\Omega}\right]_{Mott} \frac{E'^3}{E^3}\left[1 + \frac{Q^2}{2M^2}\tan^2(\theta/2)\right]$$

$$E' = \frac{E}{1 + 2\frac{E}{M}\sin^2(\theta/2)}, \quad Q^2 = 4EE'\sin^2(\theta/2)$$

The Structure of the Proton: Elastic e-p Scattering (cont.)

The proton is not a Dirac particle, having a large anomalous magnetic moment $\kappa_p = (g_p-2)/2 \approx 1.79$.

Accounting for this gives:

$$\frac{d\sigma}{d\Omega} = \left[\frac{d\sigma}{d\Omega}\right]_{Mott} \frac{E'^3}{E^3}\left[1+\kappa_p^2+\frac{Q^2}{2\mathrm{M}^2}(1+\kappa_p)^2\tan^2(\theta/2)\right]$$

Finally, if the proton has a finite charge radius, the scattering my be parameterized in terms of Form Factors $F_1(Q^2)$ and $F_2(Q^2)$,

$$\frac{d\sigma}{d\Omega} = \left[\frac{d\sigma}{d\Omega}\right]_{Mott} \frac{E'^3}{E^3}\left[F_1^2+\frac{\kappa_p^2 Q^2}{4\mathrm{M}^2}F_2^2+\frac{Q^2}{2\mathrm{M}^2}(F_1+\kappa_p F_2)^2\tan^2(\theta/2)\right]$$

The Structure of the Proton: Elastic e-p Scattering (cont.)

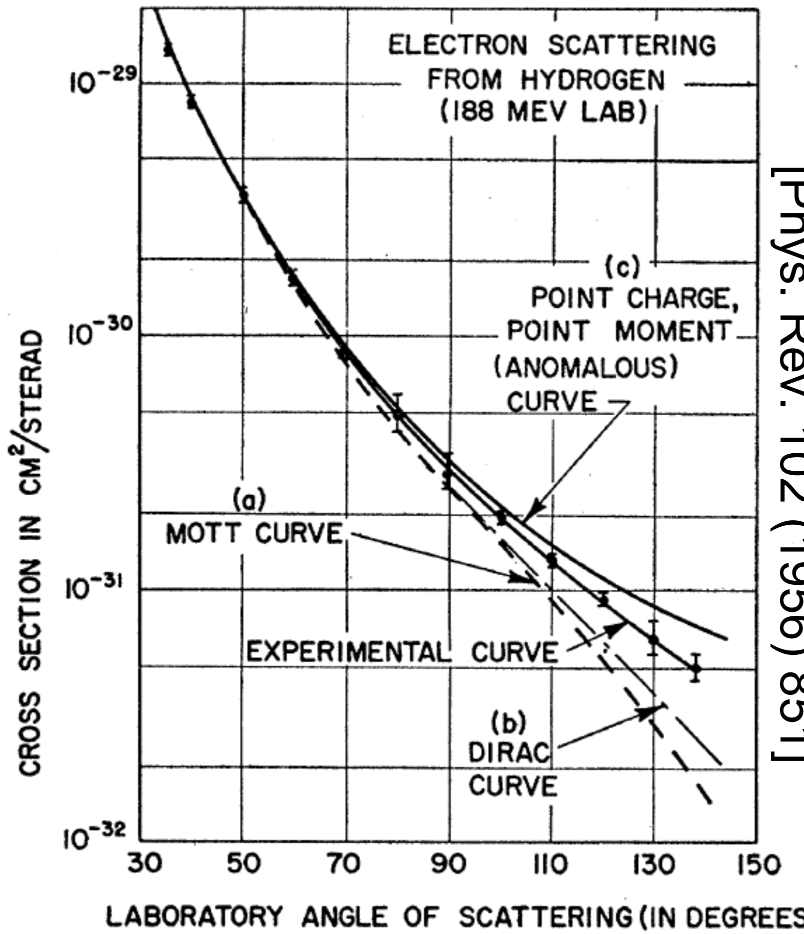
R. Hofstadter

Hofstadter and McAllister showed a clear deviation from point-like behavior. Assuming the two form factors to be equal and that

$$F(Q^2) = \int d^3r\ \rho(r) \exp(i\,\boldsymbol{r}\cdot\boldsymbol{Q})$$
$$\approx 1 - \frac{Q^2}{6}\langle r^2 \rangle + \ldots$$

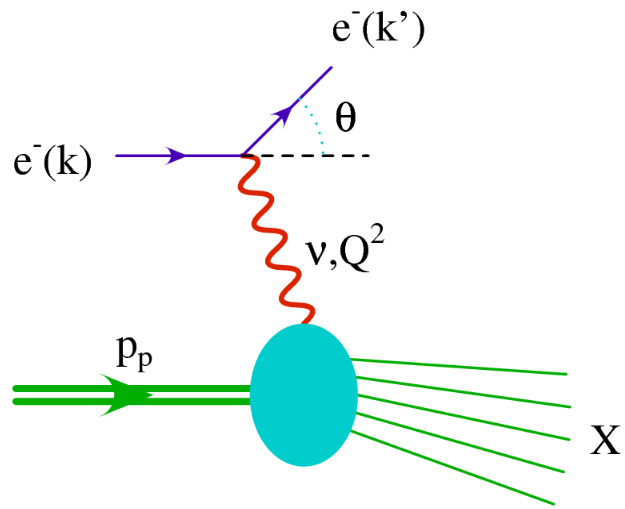
they found

$\langle r^2 \rangle^{1/2} \approx 0.74 \pm 0.24 \times 10^{-13}$cm



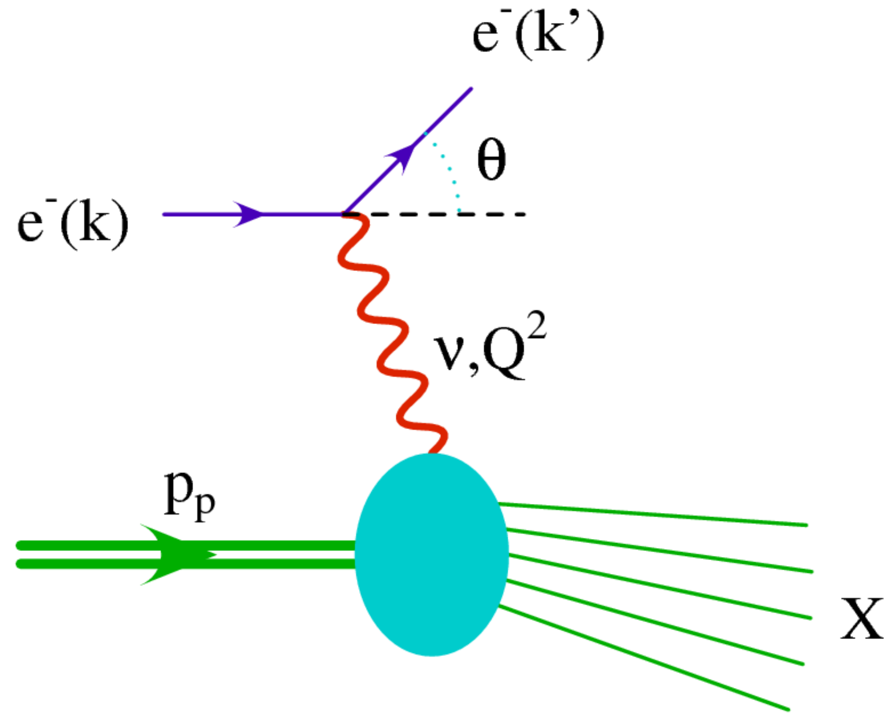
[Phys. Rev. 102 (1956) 851]

The Structure of the Proton: Deep Inelastic Scattering



With higher energy electrons, much of the scattering cross section is inelastic. Now the scattering angle and the electron energy loss are independent variables. The momentum transfer, Q^2, is determined by measuring both.

The Structure of the Proton: Deep Inelastic Scattering



The kinematics of DIS:

$e(k) + p(p) \rightarrow e(k') + X$

$q^\mu = k^\mu - k'^\mu; \; Q^2 = -q^2$

$\nu = (p \cdot q)/m_p = E_k - E_{k'}$

$x = Q^2/(2m_p\nu); \; y = (p \cdot q)/(p \cdot k)$

$W^2 = m_X^{\;2} = m_p^{\;2} + Q^2(1-x)/x$

The Structure of the Proton: Deep Inelastic Scattering

The DIS cross section may be written as a product of two tensors:

$$d\sigma = \frac{1}{2\hat{s}} \frac{d^3 k'}{k'^0} \frac{\alpha^2}{Q^4} L^{\mu\nu}(k,q)\, W_{\mu\nu}(p,q)$$

$$L^{\mu\nu}(k,q) = \frac{1}{2} Tr[\not{k}\gamma^\mu(\not{k}-\not{q})\gamma^\nu]$$

$$W_{\mu\nu}(p,q) = \frac{1}{8\pi} \sum_X \langle P(p)| j_\mu^*(0)|X\rangle\langle X|j_\nu(0)|P(p)\rangle$$

Symmetries restrict the structure of $W_{\mu\nu}$:

$$W_{\mu\nu} = -(g_{\mu\nu} - q_\mu q_\nu / q^2)\ \ F_1(x,Q^2)$$
$$+(p_\mu + x q_\mu)(p_\nu + x q_\nu)\frac{1}{m_p \nu}\ \ F_2(x,Q^2)$$

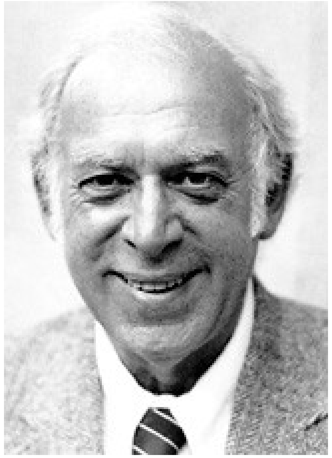
The Structure of the Proton: Deep Inelastic Scattering

From studies of current algebra, Bjorken predicted that the structure functions would become scale invariant at asymptotic energies:

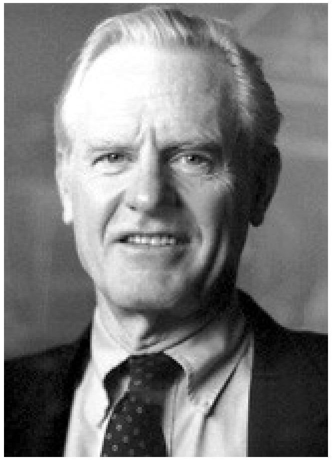
$$\lim_{\nu, Q^2 \to \infty} F_1(x, Q^2) \to F_1(x)$$

$$\lim_{\nu, Q^2 \to \infty} F_2(x, Q^2) \to F_2(x), \qquad x = \frac{Q^2}{2\mathrm{m}_p \nu}$$

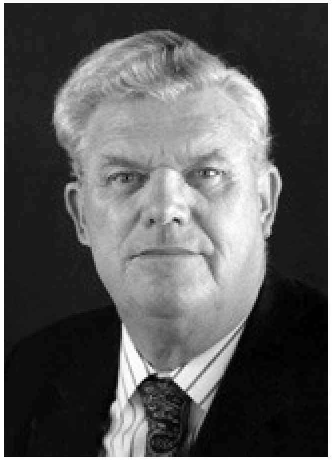
The observation of scale invariance in F_1 and F_2 was referred to as "scaling" and x was the scaling variable.



J.L. Friedman



H.W. Kendall



R.E. Taylor

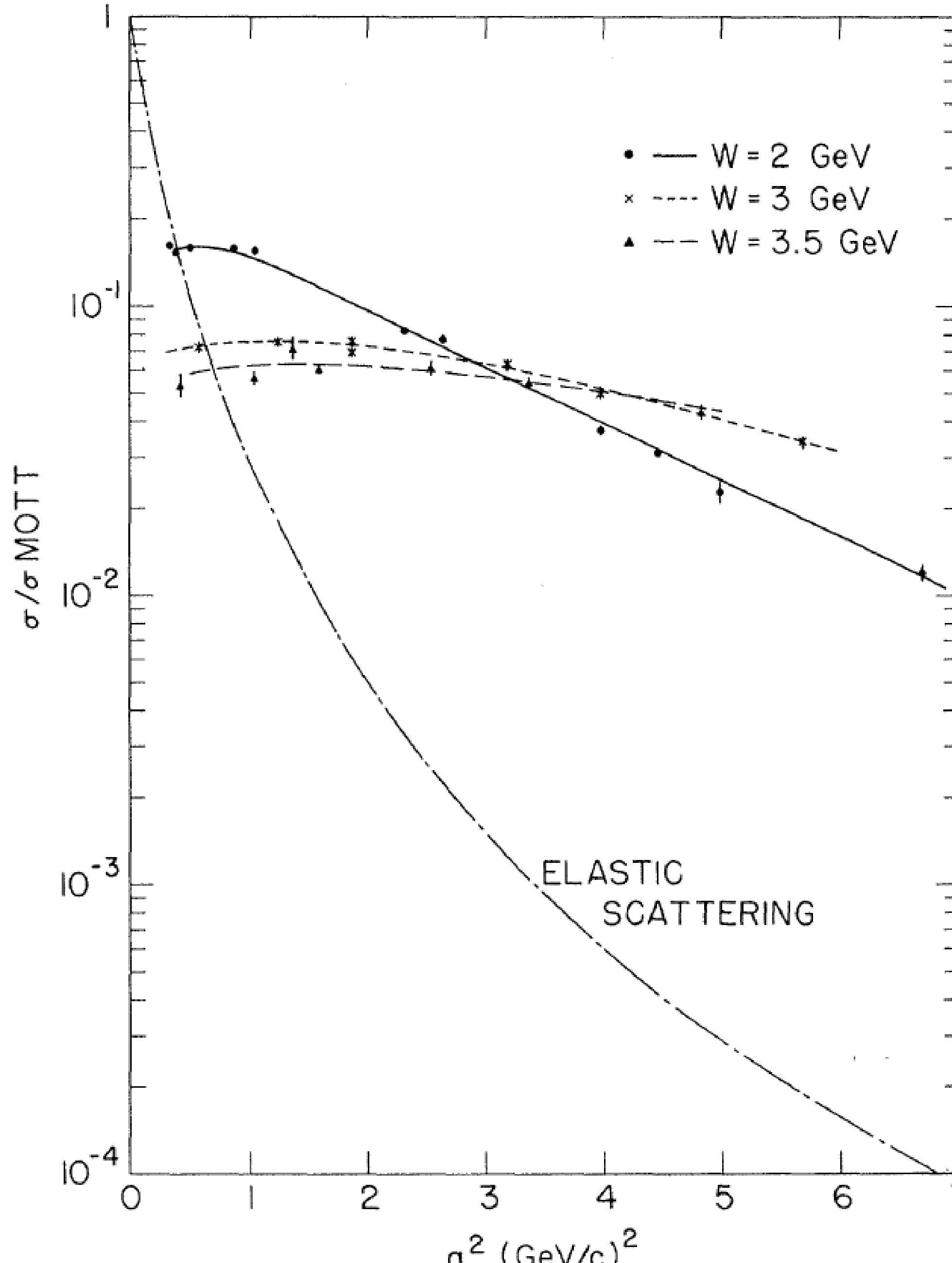
The Structure of the Proton: Deep Inelastic Scattering

In the late 1960's the linear accelerator at SLAC began producing ~20 GeV electron beams

and the SLAC-MIT experiment measured Deep Inelastic Scattering off of protons.

Q^2 Dependence of DIS

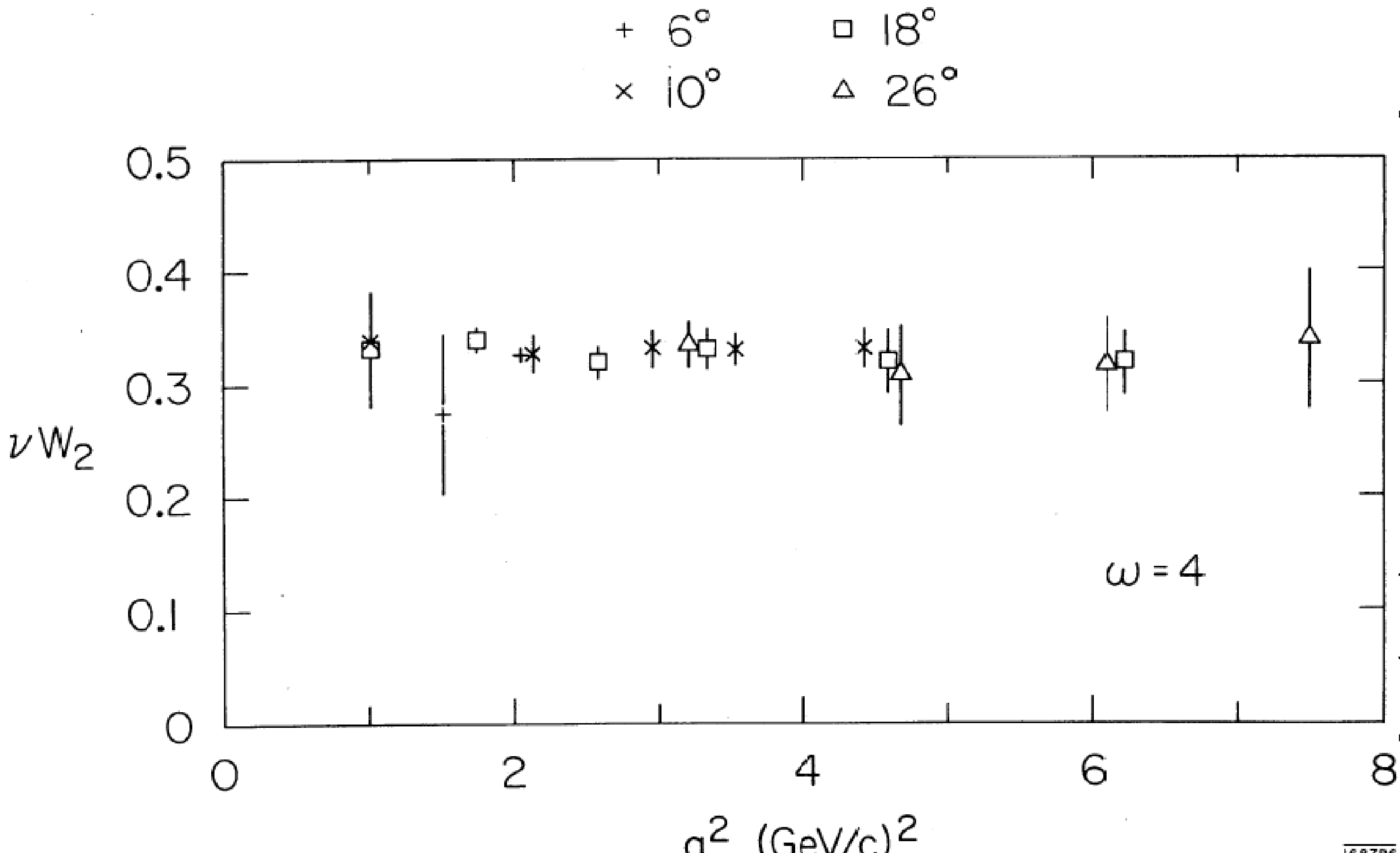
The SLAC-MIT experiment found that DIS had much weaker Q^2 dependence than expected from elastic scattering.



[Phys. Rev. Lett. 23 (1969) 935]

Scaling

The experiment also observed precocious scaling of F_2 (νW_2). Over the available range of Q^2, no scaling violation was apparent.



[Ann. Rev. Nucl. Part. Sci. 22 (1972) 203]

Explaining Scaling

Scaling could be explained by assuming that the electrons were really undergoing elastic scattering off of constituent objects in the proton that were approximately free particles!

This led to the development of the parton model.

While quarks immediately leap to mind, there were other candidates for the constituents.

And there was still no theory of what bound the partons inside of the hadron.

The Parton Model

R.P. Feynman

"We shall ... think of the incoming proton [of momentum P] as a box of partons sharing the momentum and practically free." -- R.P. Feynman

The parton model assumes that the proton wave function is dominated by momentum components below some fixed k_{max}. As $P \to \infty$, $k_3 \to xP$, $k_\perp < k_{max}$.

Parton "states" would have a lifetime $\tau \sim xP/k_\perp^2$.

An impulse $Q^2 \gg 1/\tau$ would see an essentially free parton. Asymptotic Freedom!

The Parton Model

The Parton Model is not the Quark Model! It envisioned a rich spectrum of virtual partons and anti-partons, not just three quarks rattling around.

It even included a steeply rising spectrum of "wee" partons, with very small momentum fractions, which did not contribute to DIS, but could be invoked to "explain" a variety of observations, including hadronization the rising p-p cross section, etc.

The Parton Model

The Parton Model gave a prescription for computing the DIS cross section:

$$\frac{d\sigma^{(ep)}(p,q)}{dE_{k'}\,d\Omega_{k'}} = \sum_f \int_0^1 d\xi \, \frac{d\sigma^{(ef)}_{Born}(\xi p,q)}{dE_{k'}\,d\Omega_{k'}} \phi_{f/p}(\xi) \quad ,$$

where f indicates the "flavor" of parton, ξ represents the parton momentum fraction and $\phi_{f/p}(\xi)$ is the density of parton f in the proton.

Because electromagnetic corrections are small and there was no theory of the strong interaction, the electron-parton cross section was computed in the Born approximation.

Parton Densities

The parton densities, $\phi_{f/p}(x)$ or f(x), were the number density of f-type partons in a proton with momentum fraction between x and x+dx.

Giving the partons a quark interpretation, they obeyed sum rules such that

$$Q_p = +1, \ \langle I_3 \rangle = \tfrac{1}{2}, \ S = 0.$$

$$1 = \int_0^1 dx \left\{ \frac{2}{3}[u(x) - \bar{u}(x)] - \frac{1}{3}[d(x) - \bar{d}(x)] - \frac{1}{3}[s(x) - \bar{s}(x)] \right\}$$

$$\frac{1}{2} = \frac{1}{2}\int_0^1 dx \left\{ [u(x) - \bar{u}(x)] - [d(x) - \bar{d}(x)] \right\}$$

$$0 = \int_0^1 dx [s(x) - \bar{s}(x)]$$

Parton Densities (cont.)

These equations have the solution:

$$\int_0^1 dx[u(x)-\bar{u}(x)] = 2, \; \int_0^1 dx[d(x)-\bar{d}(x)] = 1, \; \int_0^1 dx[s(x)-\bar{s}(x)] = 0$$

which is exactly that expected from the nonrelativistic quark model.

The momentum sum rule did not work. Measuring

$$f_p = \int_0^1 x\;dx\;\frac{4}{9}[u(x)+\bar{u}(x)]+\frac{1}{9}[d(x)+\bar{d}(x)]+\frac{1}{9}[s(x)+\bar{s}(x)]\approx 0.18$$

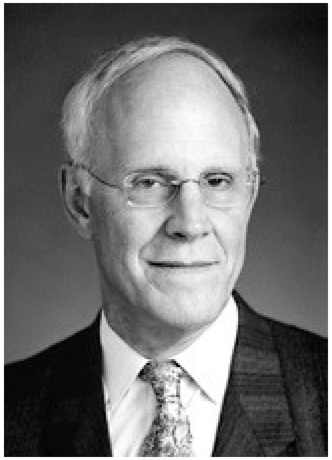
$$f_n = \int_0^1 x\;dx\;\frac{1}{9}[u(x)+\bar{u}(x)]+\frac{4}{9}[d(x)+\bar{d}(x)]+\frac{1}{9}[s(x)+\bar{s}(x)]\approx 0.12$$

and assuming

$$\int_0^1 x\;dx\;[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)+s(x)+\bar{s}(x)] = 1.0$$

implied the strange sea carried $\frac{3}{4}$ of the proton's momentum. There had to be a gluon!

The Spin of the Parton



D.J. Gross

$F_1(x)$ couples to transverse photons, σ_T

while $F_2(x)$ couples to both σ_T and σ_L

$$\sigma_T(x) \propto F_1(x), \qquad \sigma_L(x) \propto (F_2(x) - 2x F_1(x))$$

Callan and Gross showed that if the partons have spin 0 or 1

$$\lim_{Q^2 \to \infty} \sigma_T(x) \to 0 \quad \Rightarrow \quad \lim_{Q^2 \to \infty} F_1(x) = 0$$

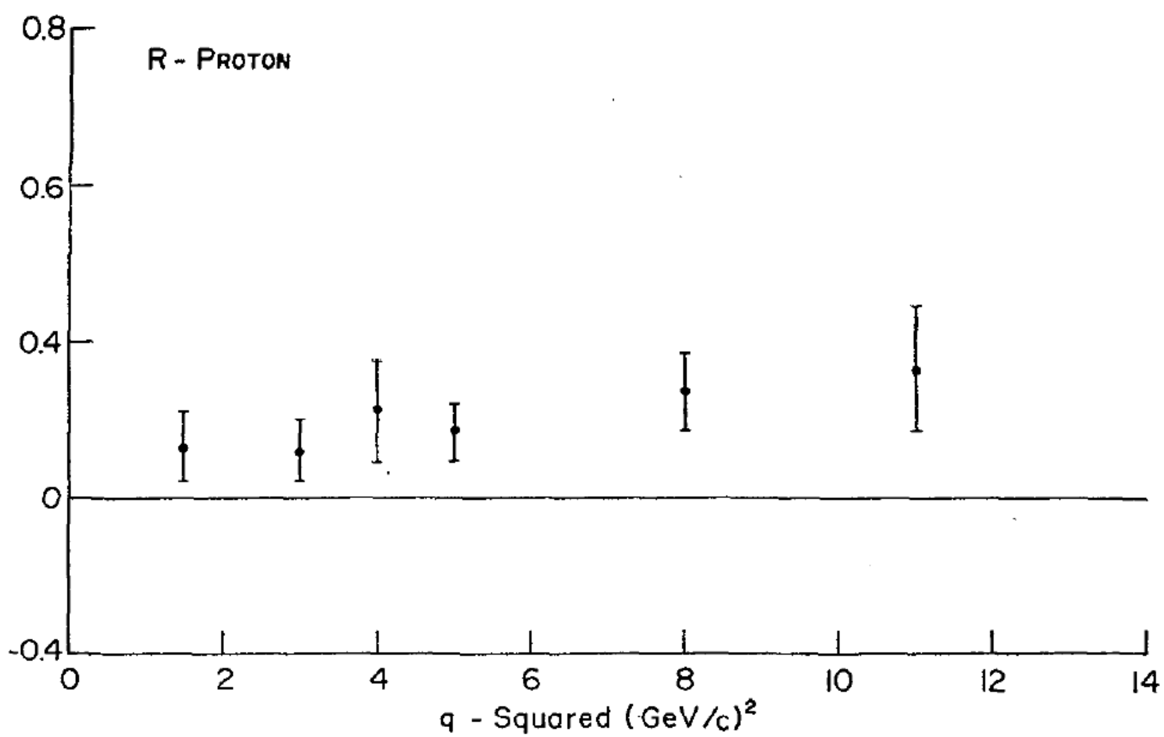
but if they have spin ½, then

$$\lim_{Q^2 \to \infty} \sigma_L(x) \to 0 \quad \Rightarrow \quad \lim_{Q^2 \to \infty} F_2(x) = 2x \lim_{Q^2 \to \infty} F_1(x)$$

This last expression, $F_2(x) = 2x F_1(x)$ is called the Callan-Gross Relation.

The Spin of the Parton

Defining R $\equiv\ \sigma_L/\sigma_T$,



[Ann. Rev. Nucl. Part. Sci. 22 (1972) 203]

SLAC-MIT found ‹R› ≈ 0.18 in the region measured, strongly favoring spin ½ and the quark interpretation.

Extending the Parton Model

Given its success in describing DIS, it was natural to try to extend the parton model to describe other high energy hadronic processes, including $e^+e^- \to$ hadrons (Drell,Yan,Levy) and $pp \to \gamma^* + X$ (Drell,Yan).

The ratio $R = \sigma_{ee\to had}/\sigma_{ee\to\mu\mu}$ (at leading order) has a simple interpretation in terms of the number and charges of constituent flavors.

The Drell-Yan process needs the parton densities and high energy. Initial studies didn't seem to work well.

Summary: The Parton Model

Studies of hadron structure showed first that the proton is a diffuse object and then later that it is a composite object.

The Parton Model was invented to describe the scaling of the structure functions of DIS.

The Parton Model gave a formula for computing DIS cross sections which could be extended to cover other hadronic processes.

Contact with the quark model comes from the spin of the charged partons and parton density sum rules which imply that the proton is (uud)


C.N. Yang

Non-Abelian Gauge Theory

Despite the advances that came from the Quark and Parton Models, there was still no fundamental theory of the strong interactions. The answer was found in an unlikely place: non-Abelian gauge theory.

Non-Abelian gauge theory was invented by Yang and Mills, who tried to develop a theory of the strong interactions by gauging isospin symmetry.

It was a beautiful theory, but it didn't work.

Non-Abelian Gauge Theory

Non-Abelian gauge theories didn't work in a variety of ways:

Gauge bosons must be massless, but "clearly" they weren't, or they would have been seen.
This problem arose in the original Yang-Mills paper.

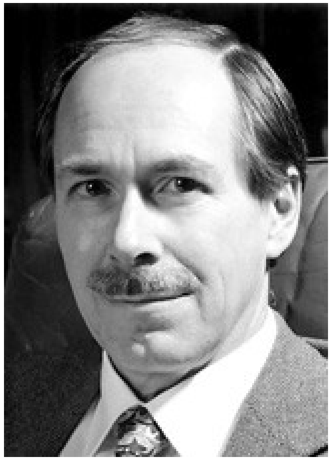
Gauge boson self-interactions made calculations cumbersome and obscured renormalizability.
In the modern view, renormalizability is not essential for an "effective field theory", but at that time, non-renormalizability presented a barrier to serious calculation.

Failures of Non-Abelian Theories

Hadronic physics is full of broken symmetries (isospin, flavor SU(3), ...) so some tried to explain the strong force with spontaneously broken gauge theories.

S. Weinberg

Weinberg describes failing to describe the strong force with a broken $SU(2)\otimes SU(2)$ symmetry (with the ρ and a_1 as the gauge bosons) when he realized that spontaneously broken $SU(2)\otimes U(1)$ would work for the weak interactions.


G. 't Hooft

Renormalizability Proven

Many theorists had given up on gauge theories, and even field theories, as viable descriptions of particle physics.

The spectacular success of QED was seen as an anomaly amidst widespread failure.

The tide began to turn in 1971 when 't Hooft demonstrated that non-Abelian gauge theories (unbroken and spontaneously broken) are renormalizable. There was a surge of interest in gauge theories and the machinery of renormalization was quickly developed.

M. Gell-Mann

QCD Proposed

In 1972, at the International Conference in Chicago, co-hosted by the new NAL, Gell-Mann and Fritzsch proposed an SU(3) gauge theory of colored quarks and gluons, as a theory of the strong interactions. That is: QCD.

They still felt that quarks and gluons were "fictitious", and were quite apologetic about constructing a theory from such objects. It would be some time before quarks were fully accepted as physical quanta.

II. FICTITIOUS QUARKS AND "GLUONS" AND THEIR STATISTICS

We assume here that quarks do not have real counterparts that are detectable in isolation in the laboratory – they are supposed to be permanently bound inside the mesons and baryons. In particular, we assume that they obey the special quark statistics, equivalent to "para–Fermi statistics of rank three" plus the requirement that mesons always be bosons and baryons fermions. The simplest description of quark statistics involves starting with three triplets of quarks, called red, white, and blue, distinguished only by the parameter referred to as color. These nine mathematical entities all obey Fermi–Dirac statistics, but real particles are required to be singlets with respect to the SU_3 of color,

It might be a convenience to abstract quark operators themselves, or other non–singlets with respect to color, along with fictitious sectors of Hilbert space with triality non–zero, but it is not a necessity. It may not even be much of a convenience, since we would then, in describing the spatial and temporal variation of these fields, be discussing a fictitious spectrum for each fictitious sector of Hilbert space, and we probably don't want to load ourselves with so much spurious information.

The ideas from this talk were not published in a journal until after the discovery of Asymptotic Freedom.

Asymptotic Freedom

The experiments in DIS placed severe constraints on a field theory of the strong interactions. Among the most difficult demands to satisfy was Asymptotic Freedom.

In a field theory, Asymptotic Freedom means that the coupling gets weaker as energy increases. To check this, one must determine the coupling's flow under the renormalization group.

$$\beta(g(\mu)) = \mu \frac{\partial g}{\partial \mu}$$

If $\beta < 0$, the theory is Asymptotically Free.

The Renormalization Group

There are many prescriptions for renormalizing a field theory. While the parameters of the theory may vary with the prescription, the physical content of the theory does not! Transformations that change the renormalization conditions, but leave the physics unchanged may be viewed as elements of a symmetry group. This is called the Renormalization Group. The parameters are said to run under the renormalization group.

The Callan-Symanzik Equation

In general, the renormalization condition can be specified in terms of a renormalization scale μ.

Consider a renormalized Green's Function

$$G^{(n)}(x_1 \ldots x_n) = \langle \Omega | T\, \phi(x_1) \ldots \phi(x_n) | \Omega \rangle_c$$

under a shift of μ: $\mu \to \mu + \delta\mu, \;\; g \to g + \delta g, \;\; \phi \to (1 + \delta\eta)\phi$

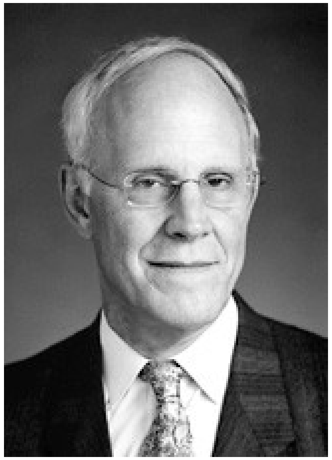
The Green's function shifts: $G^{(n)} \to (1 + n\,\delta\eta)\, G^{(n)}$.

But $G^{(n)}(x_1 \ldots x_n)$ is a function of μ and g, so

$$\frac{dG^{(n)}}{d\mu} = \frac{\partial G^{(n)}}{\partial \mu} + \frac{\partial G^{(n)}}{\partial g}\frac{\partial g}{\partial \mu} = n\frac{\partial \eta}{\partial \mu} G^{(n)}$$

Defining $\beta = \mu\, \partial g/\partial\mu$ and $\gamma = -\, \mu\, \partial\eta/\partial\mu$ gives the

Callan-Symanzik Equation: $\left[\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} + n\gamma\right] G^{(n)} = 0.$


D.J. Gross


F. Wilczek


H.D. Politzer

Asymptotic Freedom

David Gross set out to kill quantum field theory by showing that it could not describe the strong interactions. He tried to prove that:

1) Scaling demands Asymptotic Freedom.

2) No field theory is Asymptotically Free.

Politzer was interested in dynamical symmetry breaking in non-Abelian gauge theories.

Computing the β function

Using the Callan-Symanzik equation, we can compute the β function by renormalizing two and three point Green's functions.

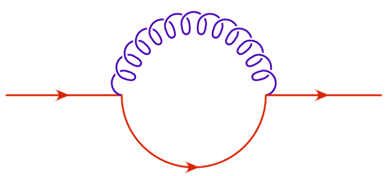
$$\mu \frac{\partial G^{(2)}(\overline{\Psi}_i, \Psi_j)}{\partial \mu} = -2\gamma_{\Psi} G^{(2)}(\overline{\Psi}_i, \Psi_j)$$

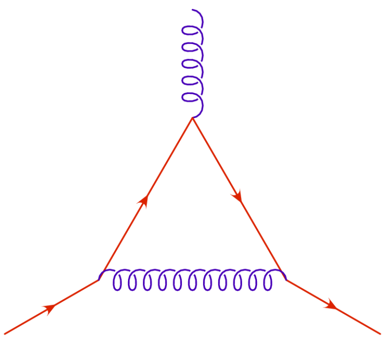
$$\mu \frac{\partial G^{(2)}(A^a_\sigma, A^b_\rho)}{\partial \mu} = -2\gamma_A G^{(2)}(A^a_\sigma, A^b_\rho)$$

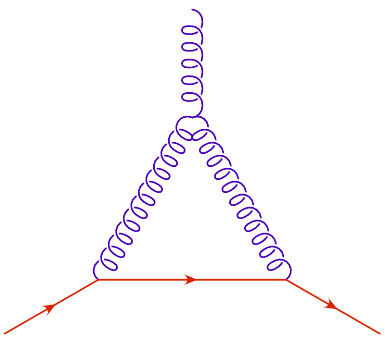
$$\beta \frac{\partial G^{(3)}(\overline{\Psi}_i, \Psi_j, A^a_\rho)}{\partial g} = -\frac{\partial G^{(3)}(\overline{\Psi}_i, \Psi_j, A^a_\rho)}{\partial \mu} - (2\gamma_\phi + \gamma_A) G^{(3)}(\overline{\Psi}_i, \Psi_j, A^a_\rho)$$

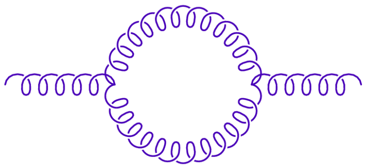
The β function and Asymptotic Freedom

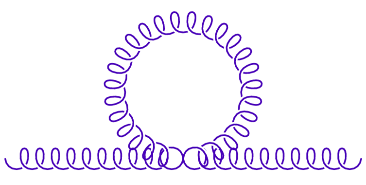
To compute the β function, one must compute loop diagrams

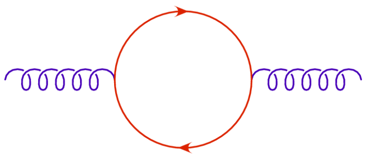


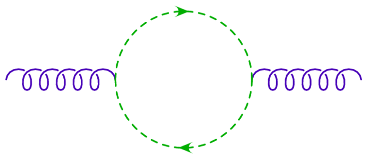




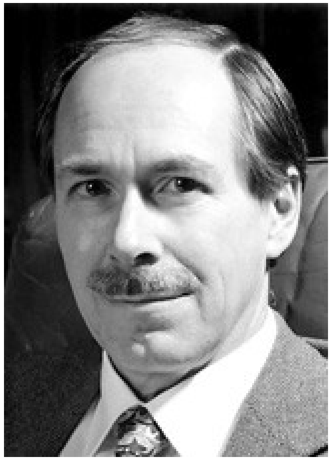




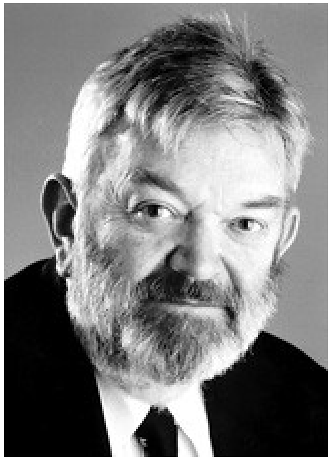




Dimensional Regularization



G. 't Hooft



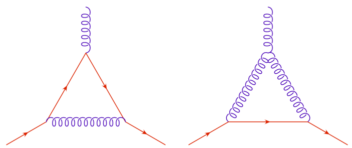
M.J.G. Veltman

Dimensional Regularization greatly simplifies loop calculations. The idea is to compute Feynman diagrams as analytic functions of the dimension of spacetime $d = 4 - 2\varepsilon$. Divergences (ultraviolet and infrared) appear as poles in ε. For physical quantities, the poles must cancel.

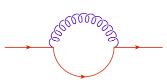
For consistency, the dimensionality of the fields and couplings change: $D[A] \rightarrow 1-\varepsilon$, $D[\psi] \rightarrow 3/2-\varepsilon$, $D[g] \rightarrow \varepsilon$. To instead keep g dimensionless, let $g \rightarrow \mu^{\varepsilon} g$ and let μ be the renormalization scale.

Computing the β function

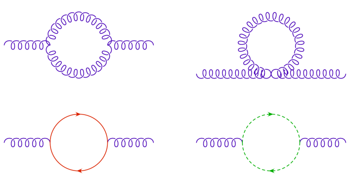
The renormalization constants are:



$$Z_{A\overline{\psi}\psi} = 1 + \frac{\alpha_s}{4\pi}(C_F + C_A)\Gamma(\epsilon)\mu^{-2\epsilon}$$



$$Z_{\psi} = 1 - \frac{\alpha_s}{4\pi} C_F \Gamma(\epsilon)\mu^{-2\epsilon}$$



$$Z_A = 1 + \frac{\alpha_s}{4\pi}\left(\frac{5}{3}C_F - \frac{4}{3}n_f T_R\right)\Gamma(\epsilon)\mu^{-2\epsilon}$$

$$\Gamma(\epsilon)\mu^{-2\epsilon} = \frac{1}{\epsilon} - 2\ln(\mu) + \ldots$$

$$\mu \frac{\partial G^{(3)}(\overline{\psi},\psi,A)}{\partial \mu} = \frac{\partial Z_{\overline{\psi}\psi A}}{\partial \ln(\mu)}, \quad \gamma_{\psi} = -\frac{1}{2}\frac{\partial Z_{\psi}}{\partial \ln(\mu)}, \quad \gamma_A = -\frac{1}{2}\frac{\partial Z_A}{\partial \ln(\mu)}$$

The result is that non-Abelian gauge theories are asymptotically free: $\beta(g(\mu)) = -g\frac{\alpha_s}{\pi}\frac{11N_c - 2n_f}{12} + \cdots$

Quantum ChromoDynamics

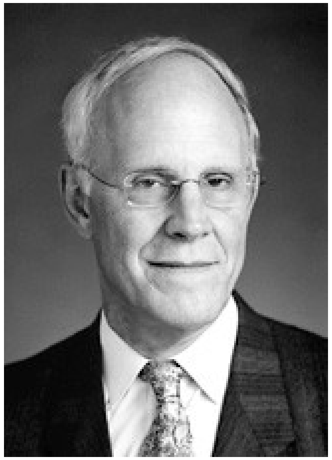
Following the discovery of Asymptotic Freedom, there was a rush to propose gauge theories of the strong interactions.

S. Weinberg

Weinberg proposed QCD as an element of a product group of gauge interactions of the Strong, Weak and Electromagnetic Forces. He noted that the strong force would conserve P and S and suggested that confinement could result from infrared slavery.

Meanwhile, Gell-Mann, Fritzsch and Leutwyler finally published their model.

Quantum ChromoDynamics



D.J. Gross

F. Wilczek

Gross and Wilczek had suggested that non-Abelian gauge theories could be theories of the Strong force when announcing Asymptotic Freedom.

They soon wrote a pair of longer papers in which they explored non-Abelian gauge theories and their behavior under the renormalization group. They echoed Weinberg's suggestion that infrared slavery could explain the confinement of quarks and gluons.

The Reality of Quarks

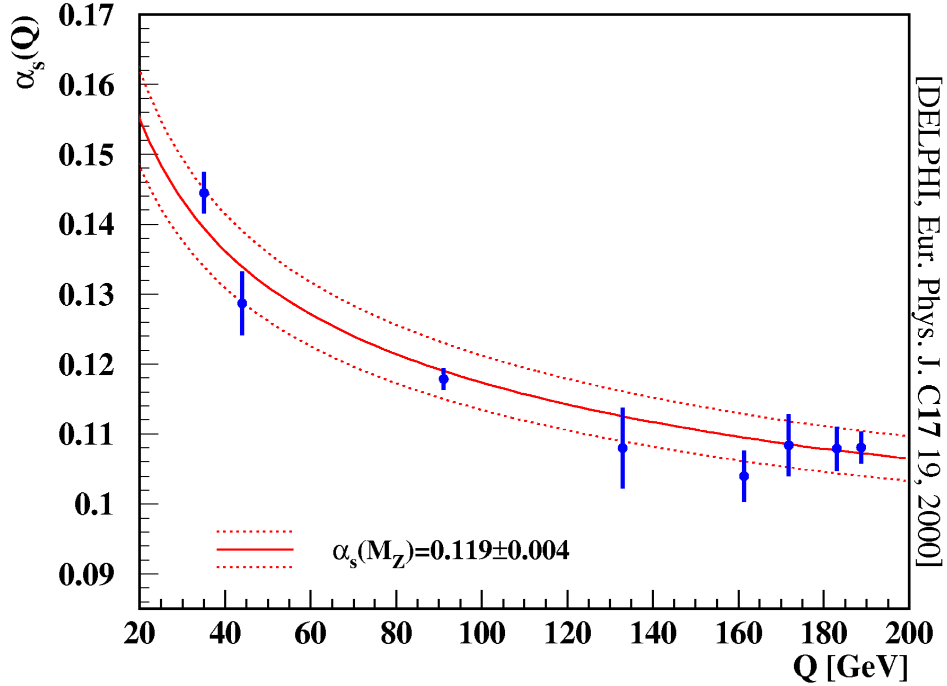
Still, the notion that quarks are "fictitious" persisted for some time. The discovery of charm helped to change this view.

Further, the study of charmonium and the description of the spectrum with an asymptotically linear potential helped to explain confinement:

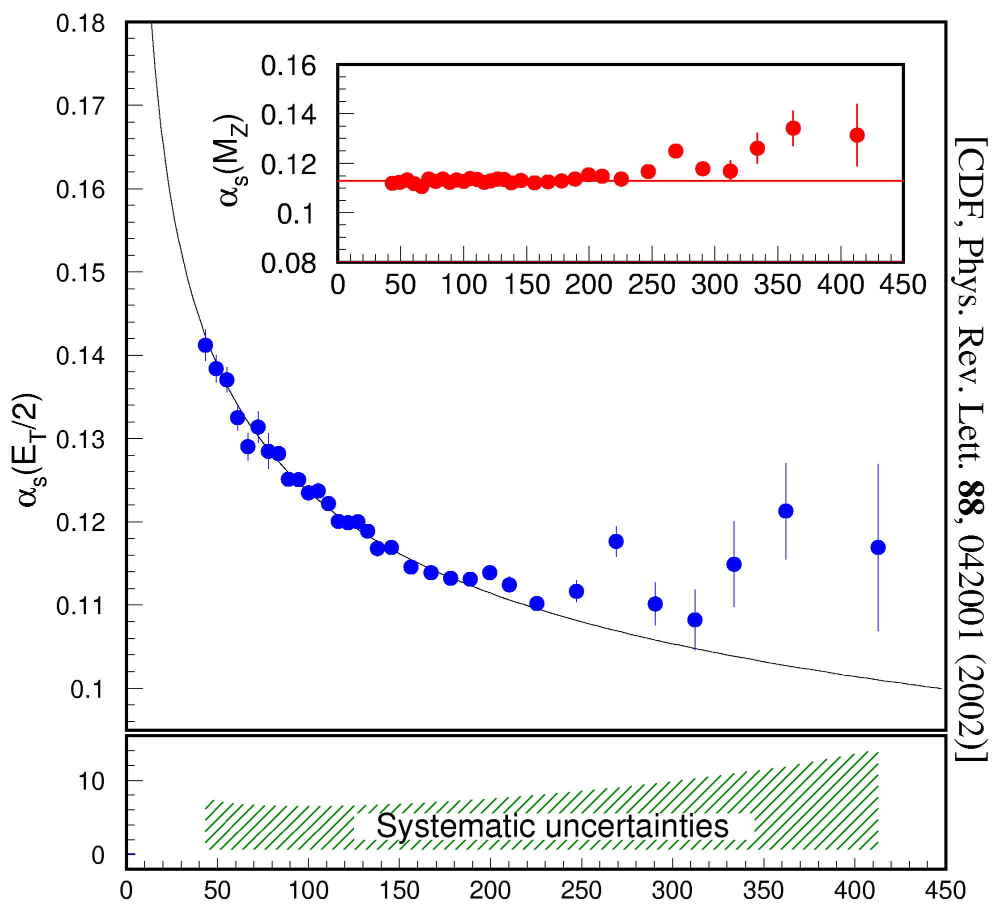
The energy in the field lines joining two separating partons continues to grow beyond the threshold for pair production, allowing each parton to neutralize it's color charge.

The Running of α_s

α_s From e^+-e^- annihilation



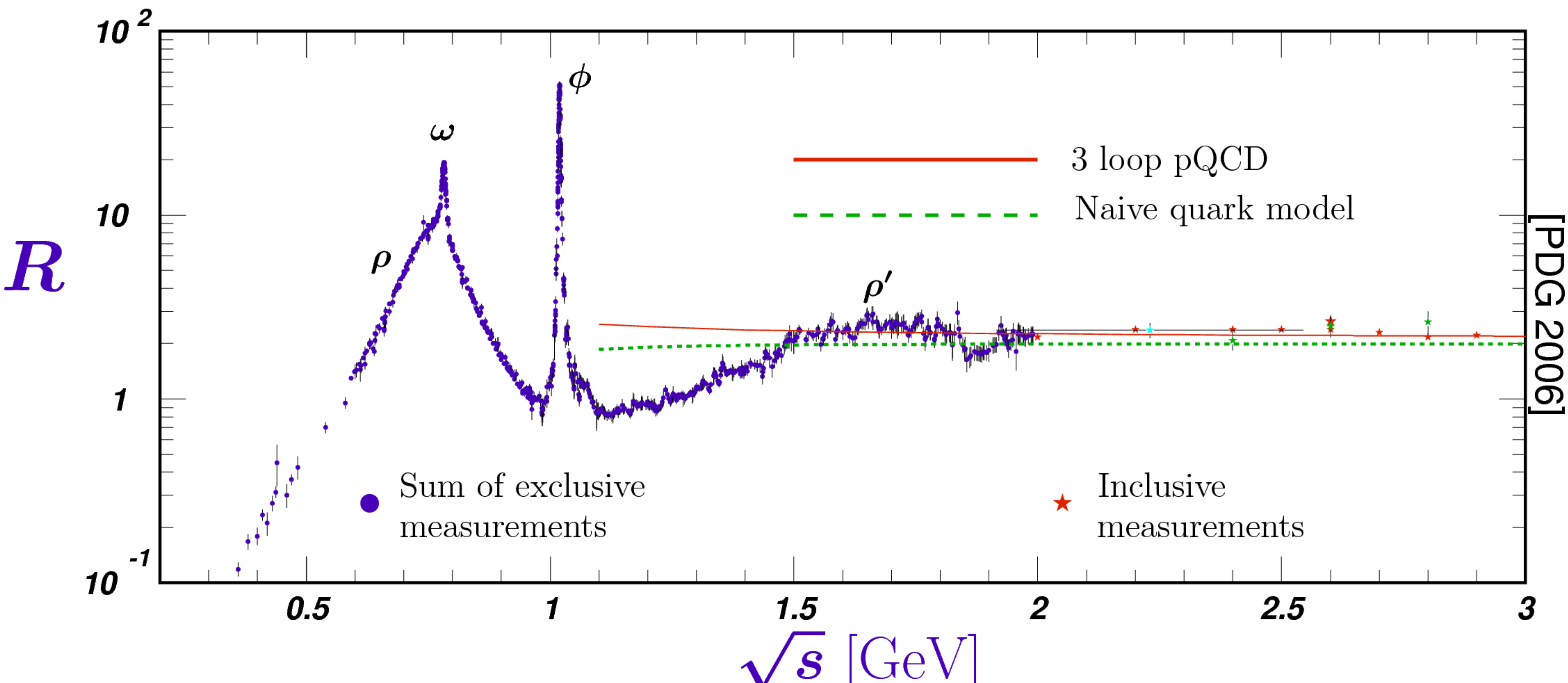
α_s From Hadronic Jets



Hints of QCD

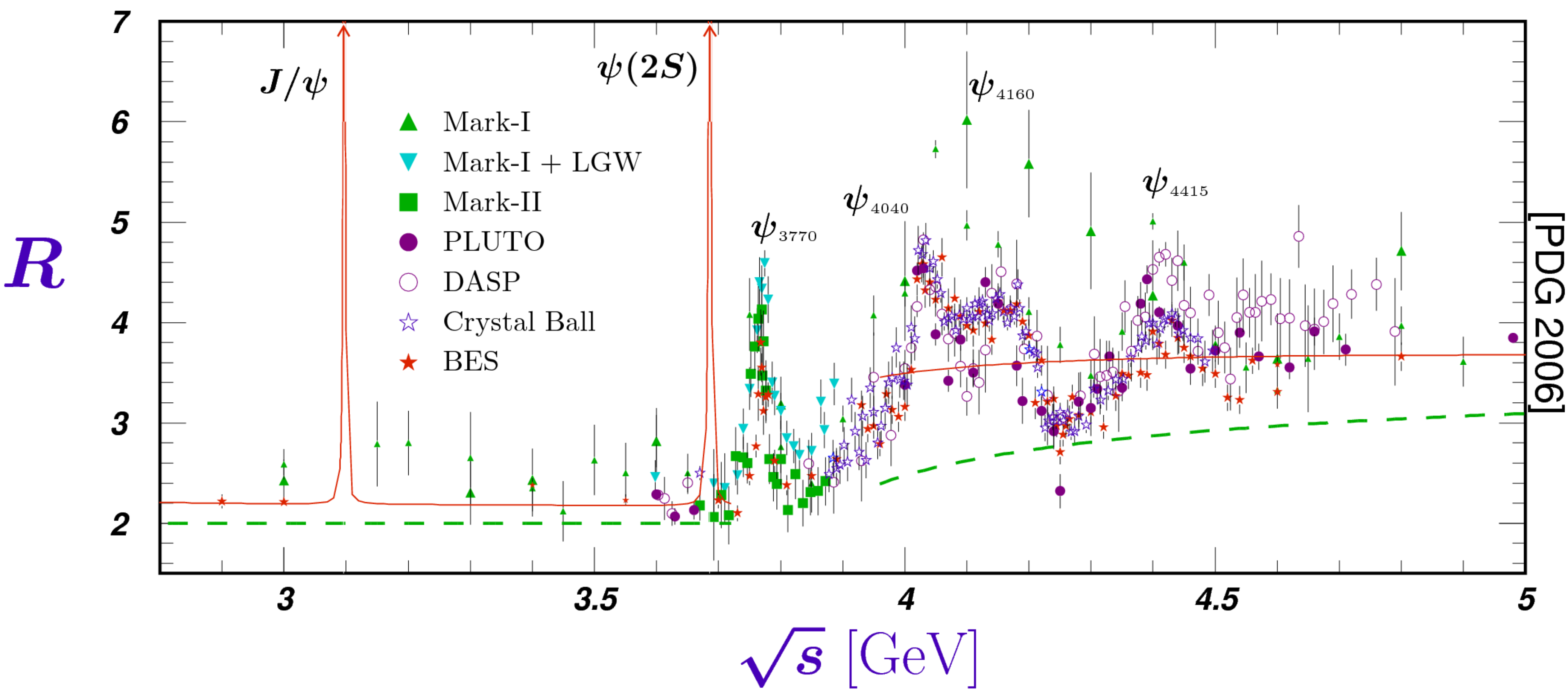
Including the color factor, the quark model gives good agreement with data (above resonance)

($R = \sigma_{had}/\sigma_{\mu\mu}$)



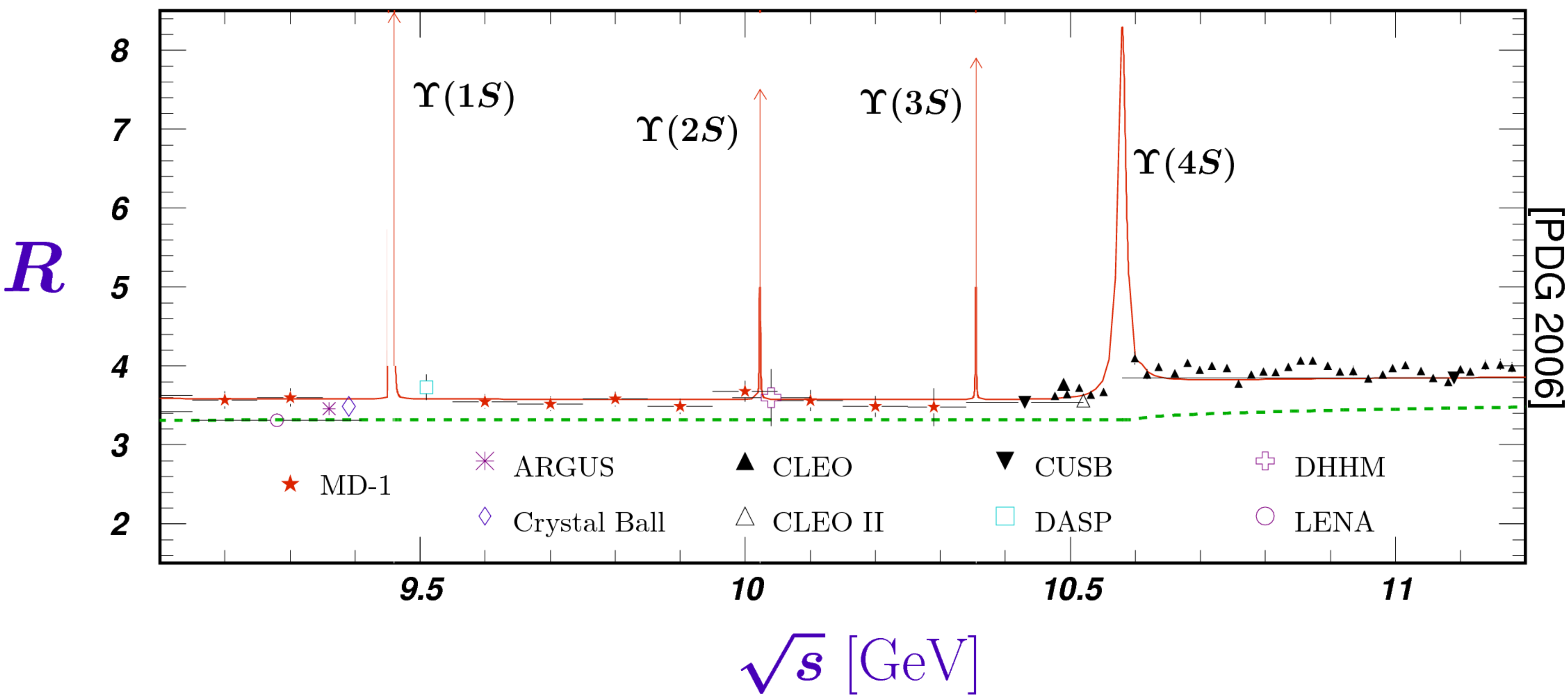
Tests of QCD

Near the J/ψ region, R seemed to disagree with QCD/parton model, but it was really charm!

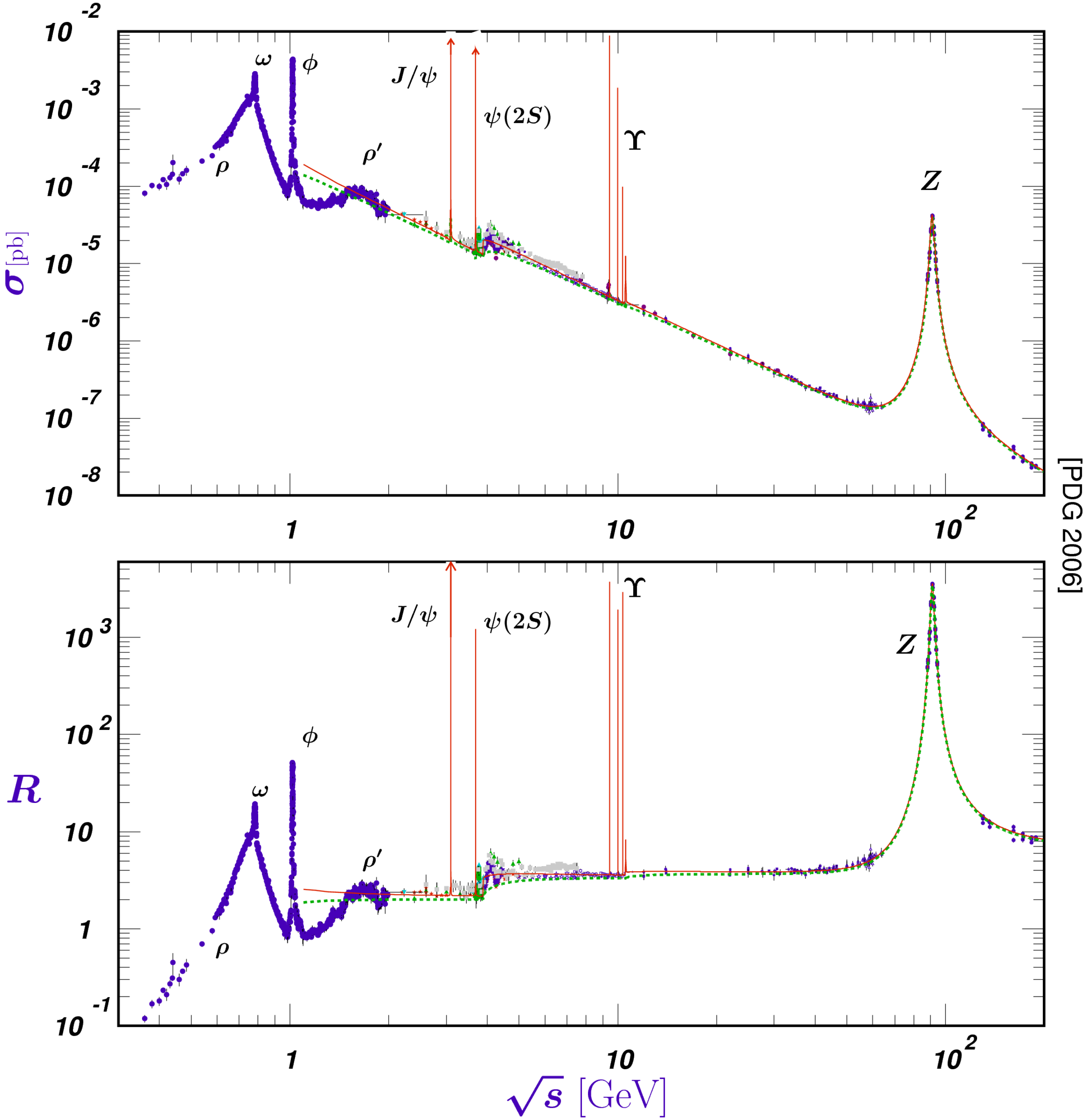


Tests of QCD

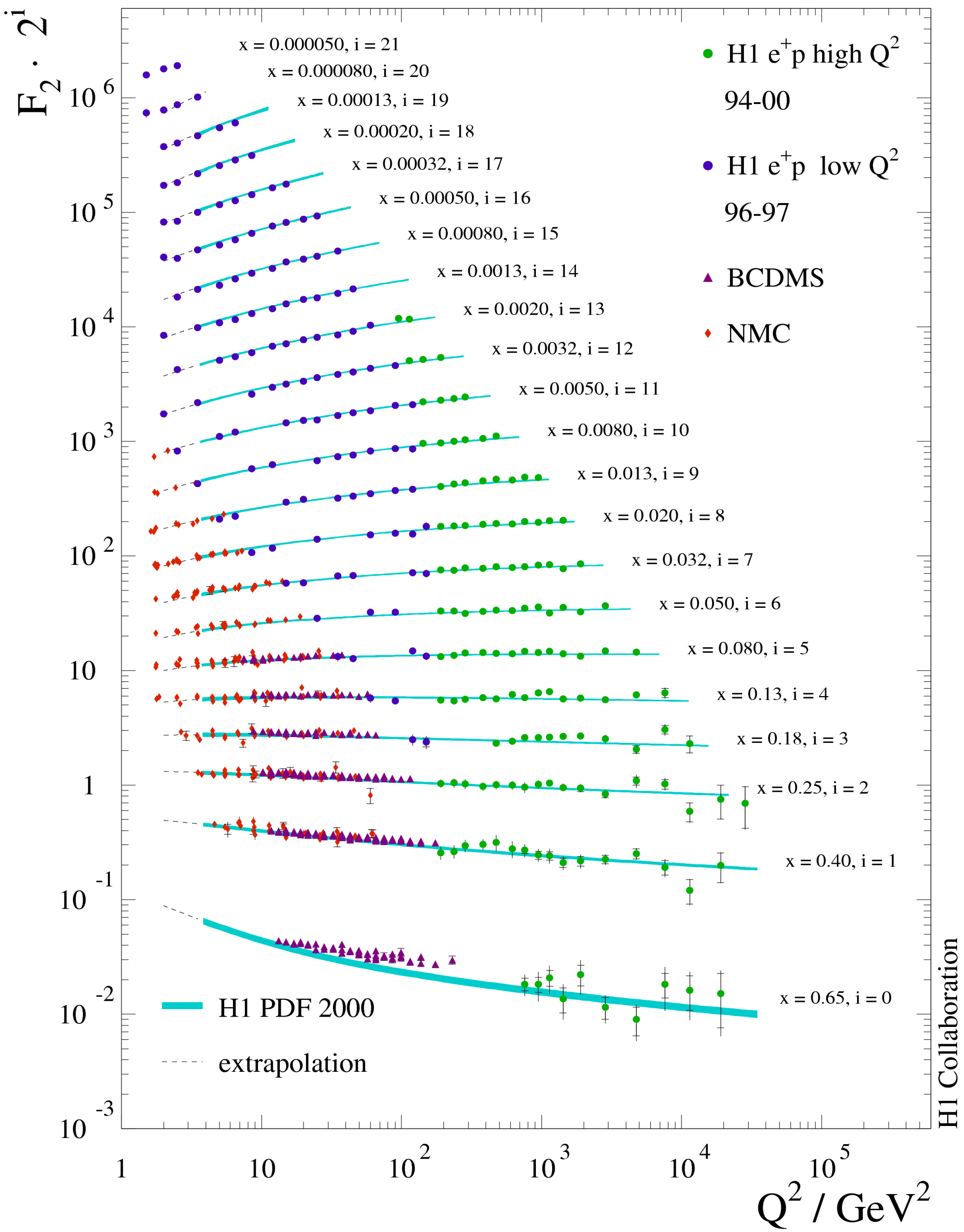
Away from resonances, pQCD works very well, even changing from 4 flavors to 5.



Tests of QCD: $e^+e^- \to$ hadrons



Logarithmic Scaling Violation



Quantum Chromodynamics

With the discovery of Asymptotic Freedom, SU(3) Yang Mills became a serious contender as a fundamental theory of the strong interactions.

All the prescriptions and hand-waving arguments of the parton model had to be made rigorous.

In particular, one needed to:

- Identify Rules for performing pQCD calculations.
- Define the parton densities.
- Derive factorization in DIS and hadron scattering.
- Specify what can be calculated.

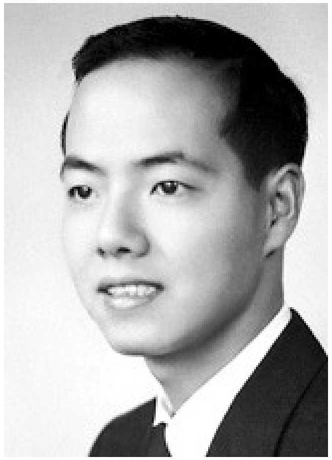
Infrared Safety

The guiding principle of applying perturbative QCD is Infrared Safety. Infrared Safe quantities do not depend on the long-distance behavior of QCD. In particular, they are finite in the limit of vanishing masses so that

$$\sigma\left(\frac{s_{ij}}{\mu^2}, \frac{m_i^2}{\mu^2}, \alpha_s(\mu)\right) = \sigma\left(\frac{s_{ij}}{\mu^2}, 0, \alpha_s(\mu)\right)\left\{1 + O\left(\frac{m_i^2}{Q^2}\right)\right\}$$

where Q^2 is a scale characteristic of the larger s_{ij}. Renormalization Group invariance then says:

$$\sigma\left(\frac{s_{ij}}{\mu^2}, 0, \alpha_s(\mu)\right) = \sigma\left(\frac{s_{ij}}{Q^2}, 0, \alpha_s(Q)\right)$$


T.D. Lee

Infrared Safety

The proof of Infrared safety comes from the KLN theorem, which states that a fully inclusive measurements, which sum over all degenerate initial and final states, are free from infrared divergences.

The short distance physics of parton scattering does not interfere with the long distance process that turns partons into hadrons.

This is why jet production is computed as parton scattering with no fragmentation. The jets contain whatever hadrons are produced.

Infrared Safety

What about less inclusive processes?

The KLN theorem can be extended to cover differential cross sections. The key is to understand the origin of infrared divergences.

Sterman showed that all infrared divergences are related to either soft or collinear momentum configurations.

As long as a measurement is "sufficiently inclusive", i.e. it sums over the soft and collinear configurations, it will be Infrared Safe!

Infrared Safety

For an operational definition of infrared safety, consider a higher order calculation:

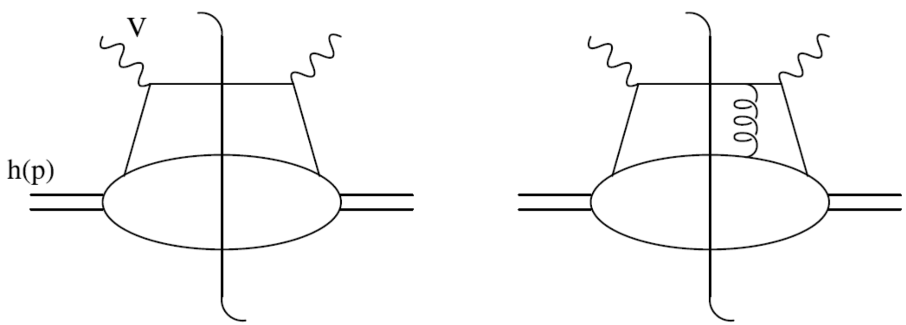
$$\sigma^{(2)}(\mathcal{J}) = \int d\Omega_n \frac{d\sigma_n^{(2)}}{d\Omega_n} \mathcal{S}_n(p_1,\ldots,p_n)$$
$$+ \int d\Omega_{n+1} \frac{d\sigma_{n+1}^{(1)}}{d\Omega_{n+1}} \mathcal{S}_{n+1}(p_1,\ldots,p_{n+1})$$
$$+ \int d\Omega_{n+2} \frac{d\sigma_{n+2}^{(0)}}{d\Omega_{n+2}} \mathcal{S}_{n+2}(p_1,\ldots,p_{n+2})$$

where $\mathcal{S}_n$ is a measurement function for observable $\mathcal{J}$. Infrared safety requires that:

$$\mathcal{S}_{n+1}(p_1,\ldots,(1-\lambda)\,p_n,\lambda\,p_n)=\mathcal{S}_n(p_1,\ldots,p_n), \qquad 0\leqslant\lambda\leqslant 1$$
$$\mathcal{S}_{n+1}(p_1,\ldots,p_n,0)=\mathcal{S}_n(p_1,\ldots,p_n).$$

The Factorization Theorem

The Factorization Theorem (Collins,Soper,Sterman) is the field theory realization of the parton model.



For DIS, it states that:

$$F_{1,3}(x,Q^2) = \sum_{i=f,\bar{f},g} \int_0^1 \frac{d\xi}{\xi} C_{1,3}^{(i)}(x/\xi, Q^2/\mu^2, \mu_f^2/\mu^2, \alpha_s(\mu))\,\phi_{i/p}(\xi,\mu_f,\mu)$$

$$F_2(x,Q^2) = \sum_{i=f,\bar{f},g} \int_0^1 d\xi\, C_2^{(i)}(x/\xi, Q^2/\mu^2, \mu_f^2/\mu^2, \alpha_s(\mu))\,\phi_{i/p}(\xi,\mu_f,\mu)$$

Factorization in Hadron-Hadron Collisions

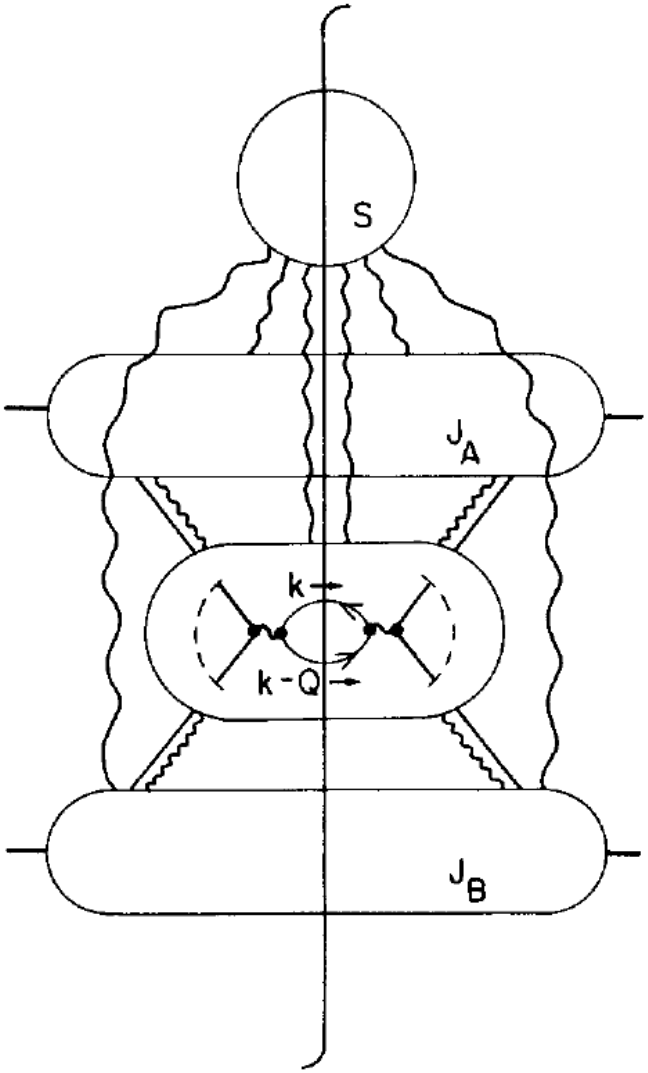
The factorization theorem also justifies the extension of the parton model to hadron-hadron collisions. Here it states:

$$\sigma(A + B \to \mathcal{J}) =$$
$$\sum_{a,b=q,\bar{q},g} \int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B \hat{\sigma}_{ab\to \mathcal{J}}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q, \mu, \alpha_s, \ldots\right) \phi_{a/A}(\xi_A, \mu)\, \phi_{b/B}(\xi_B, \mu)$$
$$+ O(1/Q^2)$$

The key departure from the simple parton model picture is that factorization works only to leading order in Q^2. At low Q^2, caveat emptor!

Factorization for Drell-Yan

A crucial piece of the theorem is that soft exchanges between the incoming hadrons cancel at the leading power of $1/Q^2$.



Power corrections at low Q^2 explain why early Drell-Yan measurements did not support the parton model .

The Factorization Theorem

The fundamental aspect of the factorization theorem is the separation of long-distance and short-distance effects. The factorization scale μ is arbitrary.

$$\sigma(A+B\to \mathcal{J}) =$$

$$\sum_{a,b=q,\bar{q},g} \int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B \, \hat{\sigma}_{ab\to \mathcal{J}}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q, \mu, \alpha_s, \ldots\right) \phi_{a/A}(\xi_A, \mu)\, \phi_{b/B}(\xi_B, \mu)$$

$$+\, O(1/Q^2)$$

All long-distance initial-state physics is contained in $\phi_{a/A}$, $\phi_{b/B}$. Short-distance physics is in $\hat{\sigma}$ and is computed in perturbation theory.

Feynman Rules for pQCD

$$\mathscr{L}_{QCD} = -\frac{1}{4} F^{a\mu\nu} F^{a}_{\mu\nu} + \bar{q}_i \, i \, \not{D}_{ij} q_j - m_q \bar{q}_i q_i - \bar{\eta}^a \partial^\mu D^{ac}_\mu \eta^c$$

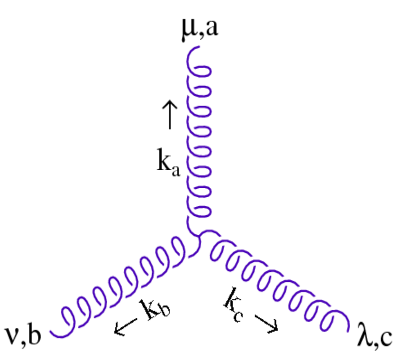
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$D^\mu_{ij} = \partial^\mu \delta_{ij} - i g t^a_{ij} A^{a\mu}, \qquad D^{ab}_\mu = \partial_\mu \delta^{ab} - g f^{abc} A^c_\mu$$

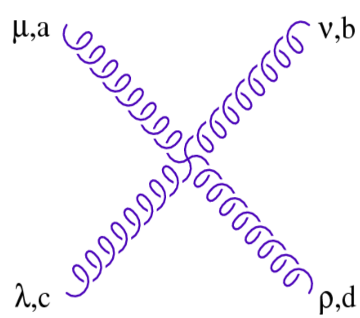
$$\frac{-i\eta^{\mu\nu}\delta^{ab}}{k^2+i\varepsilon}$$

$$\frac{i\delta^{ij}}{\not{k}+i\varepsilon}$$

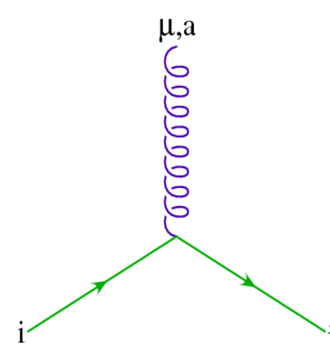
$$\frac{i\delta^{ab}}{k^2+i\varepsilon}$$



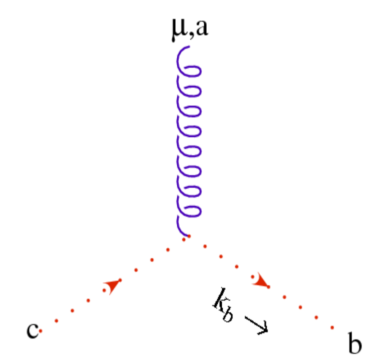
$$g f^{abc}[\eta_{\mu\nu}(k_b-k_a)_\lambda + \eta_{\mu\lambda}(k_a-k_c)_\nu + \eta_{\lambda\nu}(k_c-k_b)_\mu]$$



$$-ig^2[f^{xab}f^{xcd}(\eta_{\mu\lambda}\eta_{\nu\rho}-\eta_{\mu\rho}\eta_{\nu\lambda}) + f^{xac}f^{xbd}(\eta_{\mu\nu}\eta_{\lambda\rho}-\eta_{\mu\rho}\eta_{\nu\lambda}) + f^{xad}f^{xbc}(\eta_{\mu\nu}\eta_{\lambda\rho}-\eta_{\mu\lambda}\eta_{\nu\rho})]$$



$$ig\, T^a_{ji}\gamma_\mu$$



$$g f^{abc} k_{b\mu}$$

Parton Distributions

We would like to define parton density functions like those in the parton model. That is, for instance, $\phi_{u/p}(x)$ representing the probability of finding a u-quark in the proton with momentum fraction between x and x+dx.

Since we are now working within a fundamental theory where one can calculate radiative correction, however, we must demand a rigorous definition.

Parton Distributions (cont.)

Parton Distribution Functions are defined in terms of matrix elements of renormalized operators in QCD. For a hadron h with momentum p,

$$\phi_{q_j/h}(x,\mu) = \frac{1}{4\pi}\int dy^- e^{-ixp^+y^-}\langle p|\overline{\psi}_j(0,y^-,\mathbf{0}_T)\gamma^+ \boldsymbol{W}(y^-,0)\,\psi_j(0)|p\rangle_R$$

$$\phi_{\bar{q}_j/h}(x,\mu) = \frac{1}{4\pi}\int dy^- e^{-ixp^+y^-}\langle p|\psi_j(0,y^-,\mathbf{0}_T)\gamma^+ \boldsymbol{W}(y^-,0)\,\overline{\psi}_j(0)|p\rangle_R$$

$$\phi_{g/h}(x,\mu) = \frac{1}{4\pi}\int dy^- e^{-ixp^+y^-}\langle p|F_a^{+\nu}(0,y^-,\mathbf{0}_T)\gamma^+ \boldsymbol{W}(y^-,0)\,F_{a\nu}{}^{+}(0)|p\rangle_R$$

Where W is a Wilson line,

$$\boldsymbol{W}(y^-,0) = \boldsymbol{P}\exp[ig\int_0^{y^-} ds^- A_a^+(0,s^-,\mathbf{0}_T)\,t^a]$$

Parton Distributions (cont.)

Observations:

1) PDFs are non-perturbative.

The matrix elements involve the proton wave function. They must be extracted from measurements.

2) PDFs are Ultraviolet Singular.

Renormalization spoils the interpretation as number densities. Treated as distributions, they still satisfy the sum rules.

3) PDFs are renormalized.

They obey renormalization group equations (the DGLAP equations), and evolve in Q^2.

4) PDFs are universal.

They are process independent. PDFs determined in DIS can be used in hadron-hadron collisions.

Parton Evolution

Unlike the parton densities of the parton model, PDFs evolve in Q^2 according to DGLAP equations:

$$\mu^2 \frac{d}{d\mu^2} \phi_{a/p}(x) = (P_{ab} \otimes \phi_{b/p})(x),$$

where

$$(f \otimes g)(x) = \int_0^1 dy\, dz\, f(y)\, g(z)\, \delta(x - yz) = \int_x^1 \frac{dz}{z} f(x/z)\, g(z)$$

$$P_{ab}(x) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi} \right)^{n+1} P_{ab}^{(n)}(x)$$

The splitting functions $P_{ab}(x)$ are now known through order α_s^3.

Determining PDFs

PDFs are determined by comparing perturbative QCD calculations to experimental results.

Experiments are sensitive to different combinations of the PDFs, over differing ranges of parton momentum fraction x and are performed at a variety of values of Q^2.

The fitting procedure must take the evolution in Q^2 between experiments into account.

Fitting PDFs

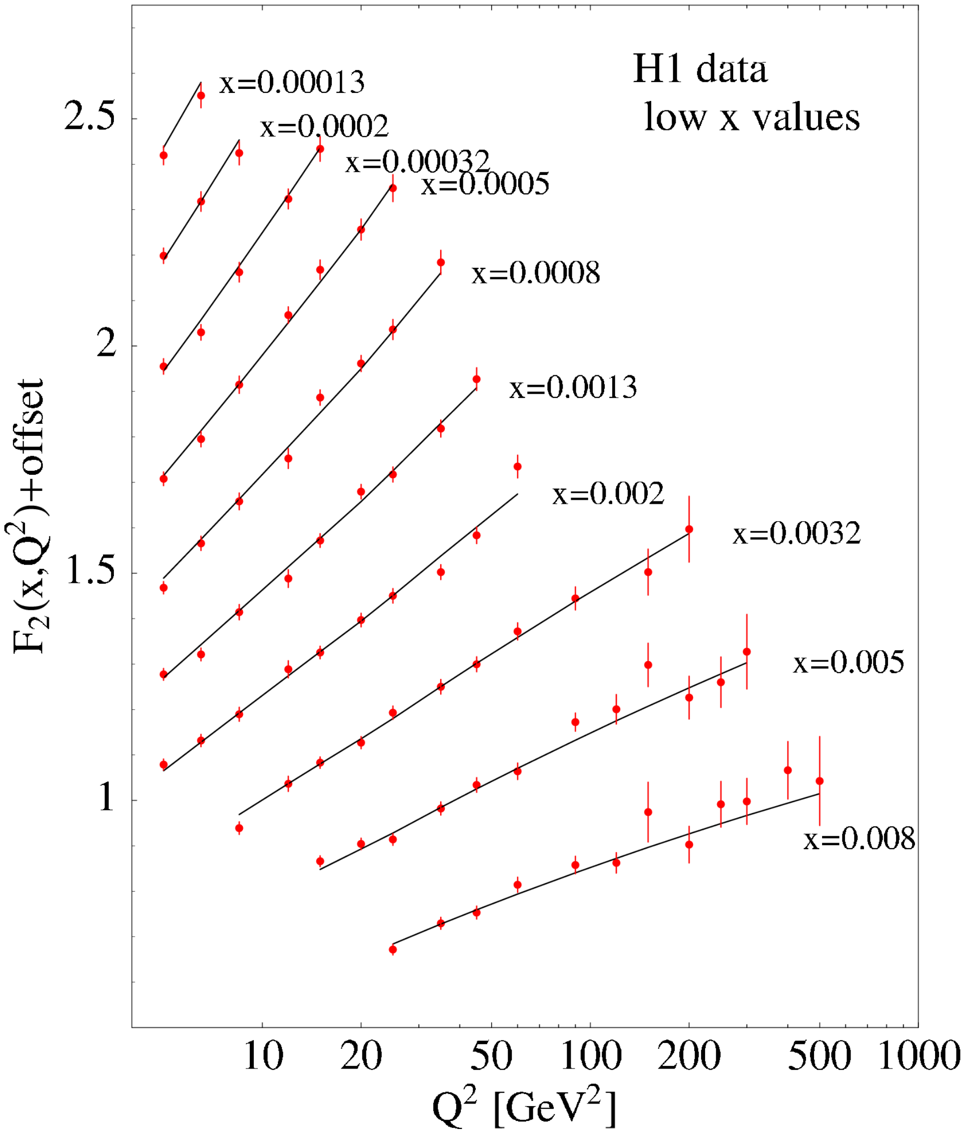
A wide variety of data are used to fit PDFs.

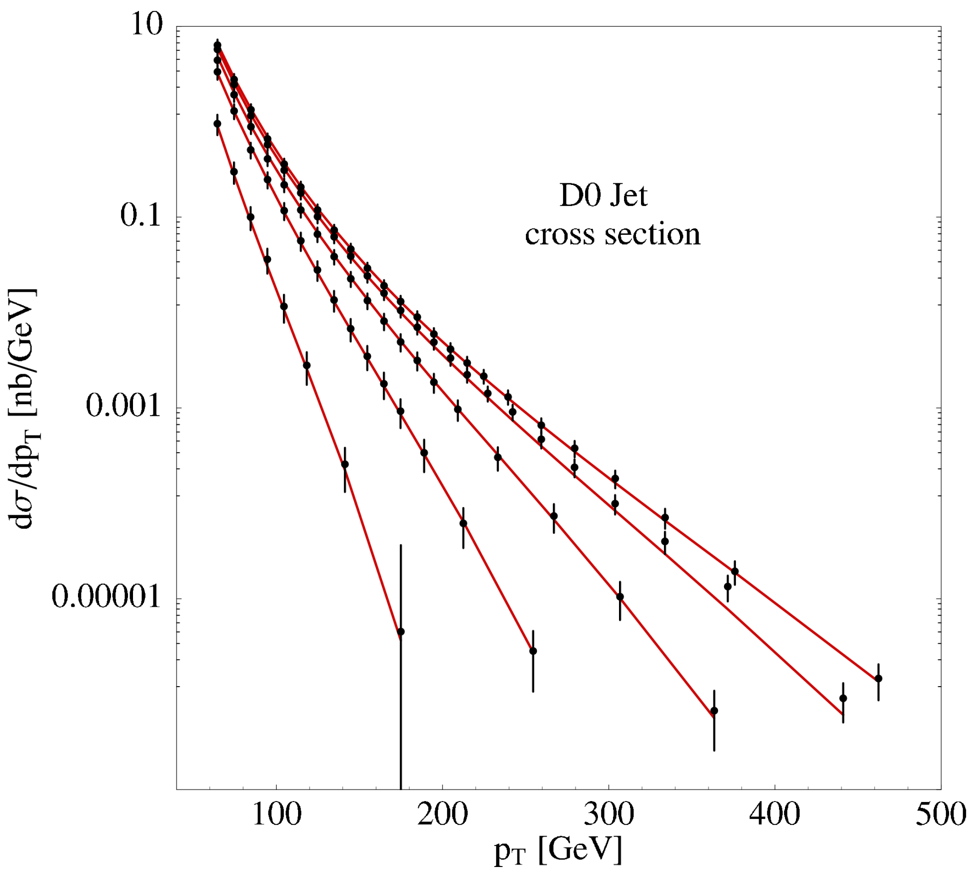
- DIS Structure Functions at H1 and ZEUS
- W (lepton) asymmetry at CDF
- Inclusive Jet Production at Tevatron
- Fixed target DIS (proton and deuteron)
- Fixed target Drell-Yan (proton and deuteron)
- Neutrino DIS (nuclear target)

The low energy data often require corrections to deal with "higher twist" effects. Deuterium and nuclear data require still more corrections.

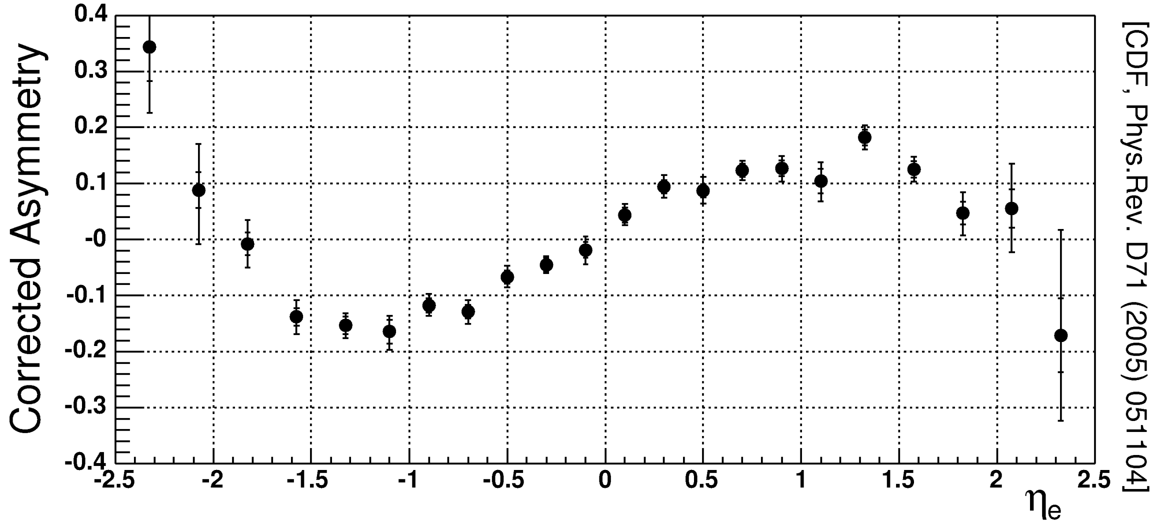
PDF Fits (cont.)

Modern PDFs fit the available data very well.

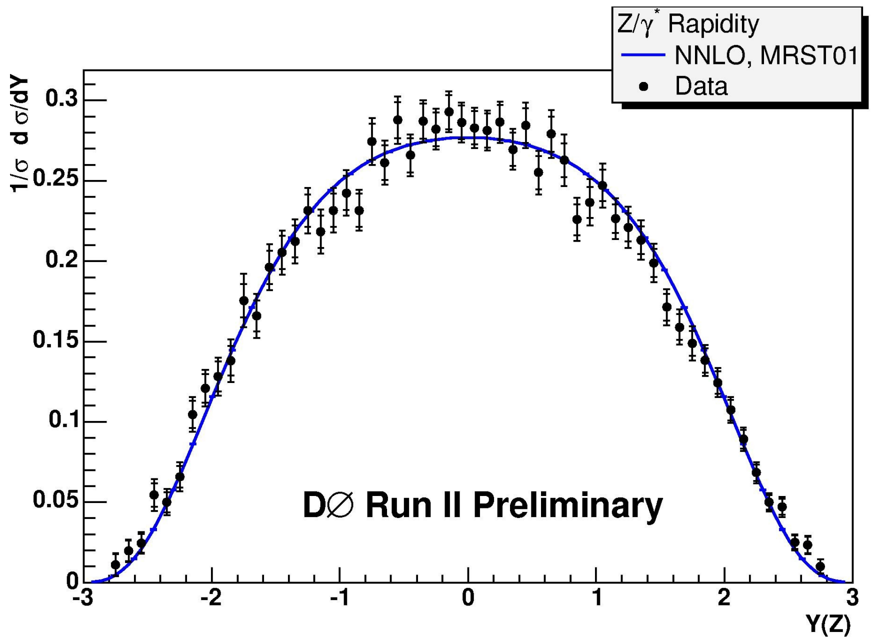




PDF Fits



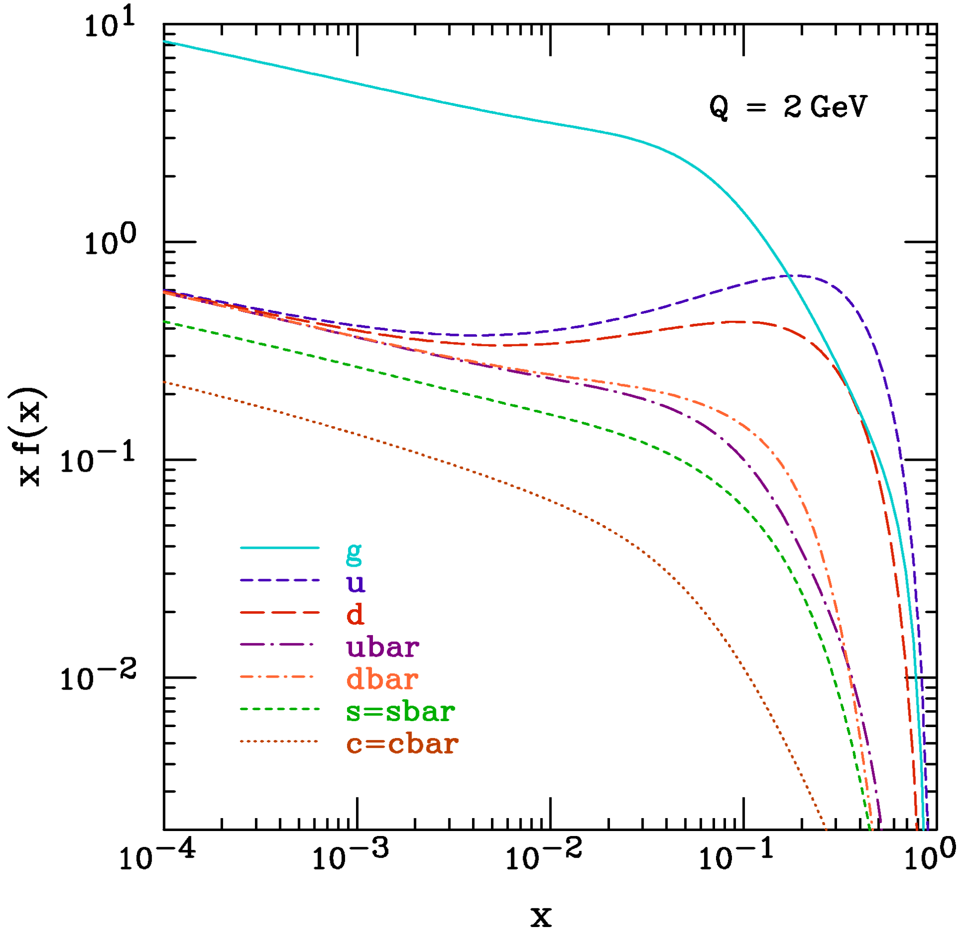
W boson charge asymmetry

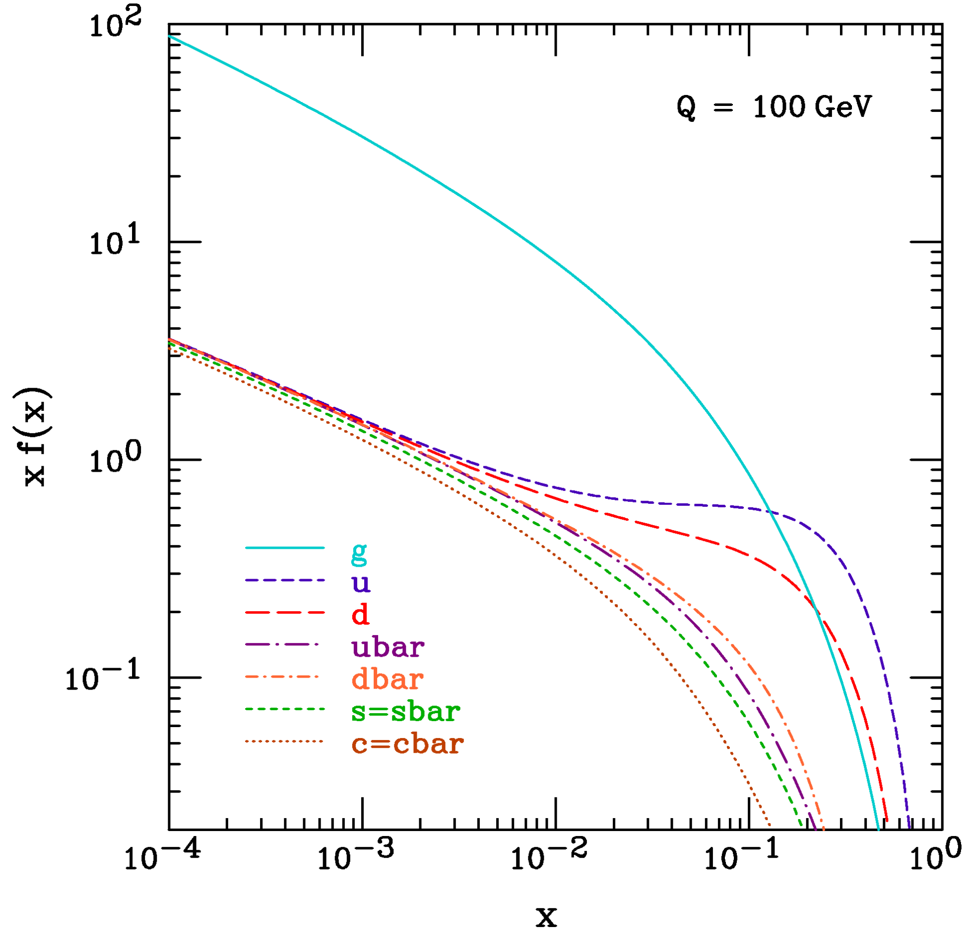


Z/γ^* rapidity distribution

PDF Fits

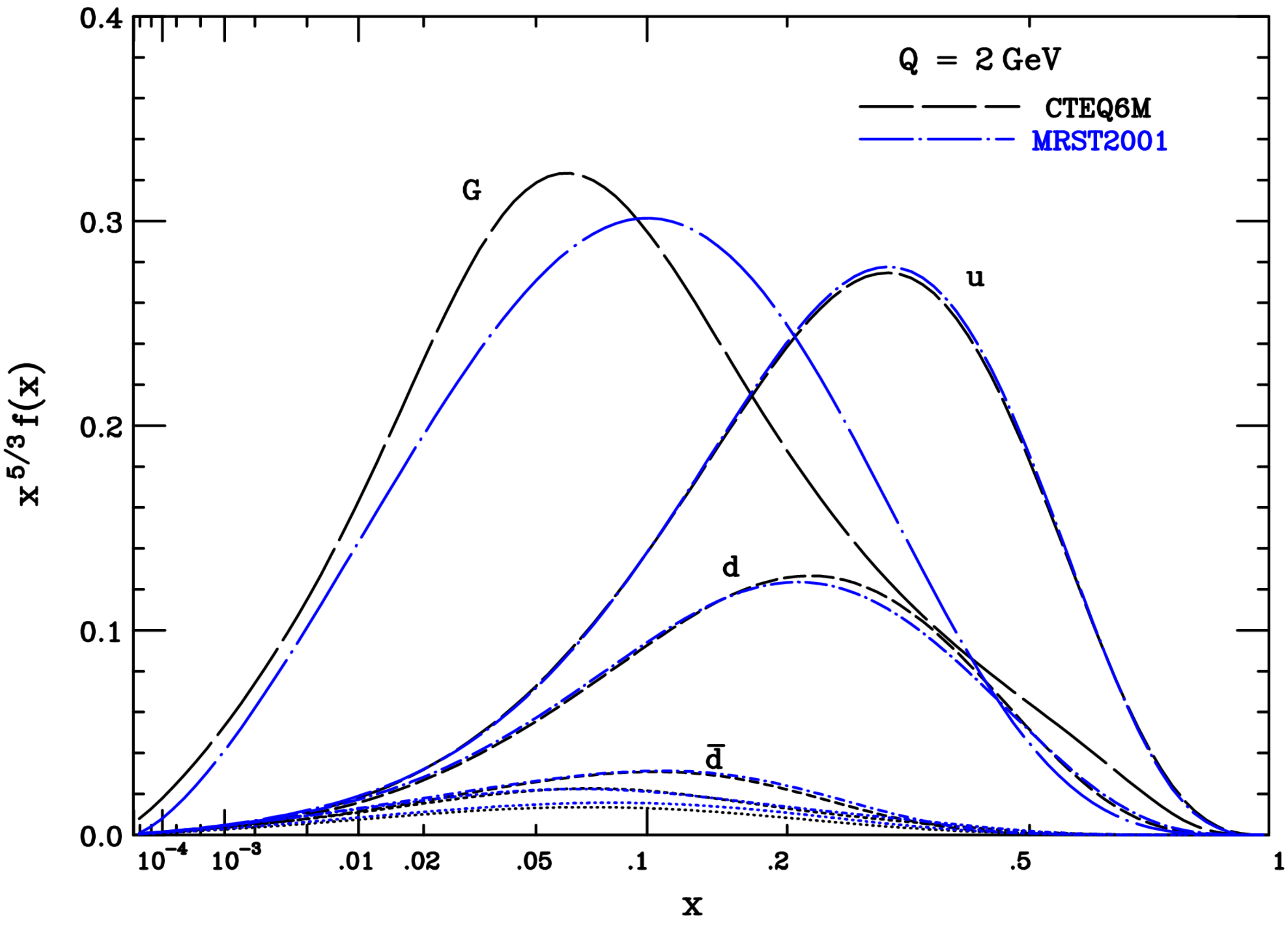
CTEQ6M at two different values of Q:





PDF Fits (cont.)

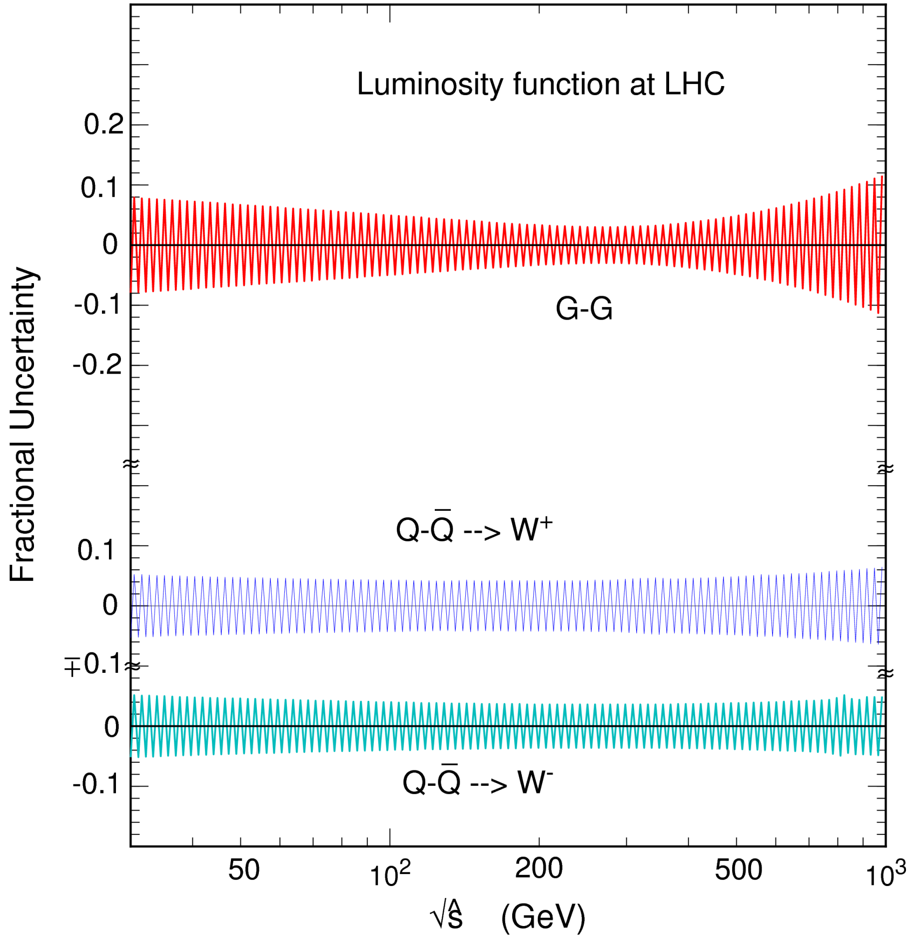
Still the gluon is hard to constrain.

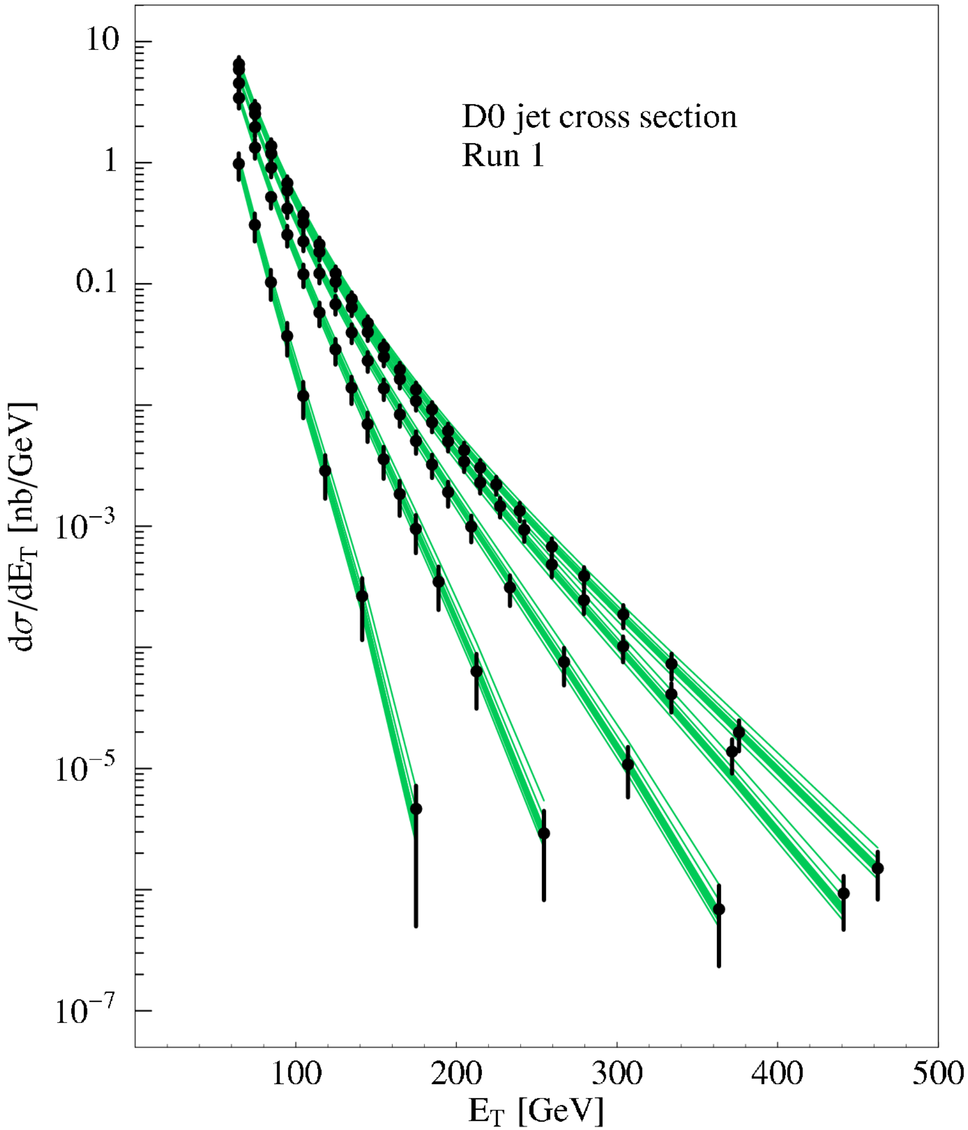


PDF Uncertainties

For many years, PDF "best fits" were distributed without any serious attempt to quantify how good the best fits were. It is now common for PDF fitters to produce sets of PDFs that map out a range of "good" fits. Averaging over the sets introduces uncertainty to Monte Carlo calculations that reflect the uncertainty in the input PDFs.

PDF Uncertainties (CTEQ6)





The Hard Scattering

The PDFs contain all of the initial state long-distance physics. The short-distance physics is contained in the hard-scattering cross section, often called the partonic cross section.

The partonic cross section is computed by using the Feynman Rules to calculate on-shell matrix elements of (usually) massless quarks and gluons, which are then integrated over the phase space of the final state partons.

Feynman Rules for pQCD

$$\mathscr{L}_{QCD} = -\frac{1}{4} F^{a\mu\nu} F^{a}_{\mu\nu} + \bar{q}_i \, i \not{D}_{ij} q_j - m_q \bar{q}_i q_i - \bar{\eta}^a \partial^\mu D^{ac}_\mu \eta^c$$

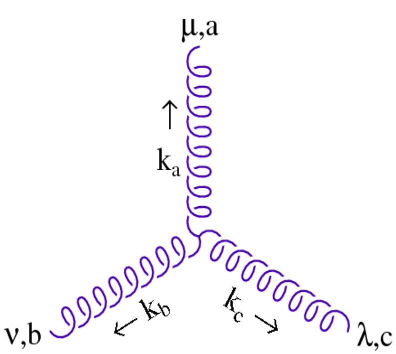
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$D^\mu_{ij} = \partial^\mu \delta_{ij} - i g t^a_{ij} A^{a\mu}, \qquad D^{ab}_\mu = \partial_\mu \delta^{ab} - g f^{abc} A^c_\mu$$

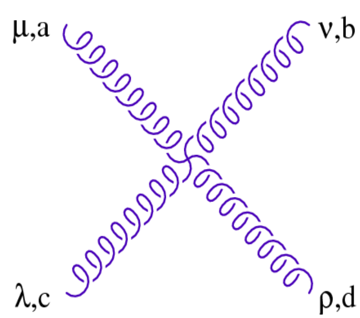
$$\frac{-i\eta^{\mu\nu}\delta^{ab}}{k^2+i\varepsilon}$$

$$\frac{i\delta^{ij}}{\not{k}+i\varepsilon}$$

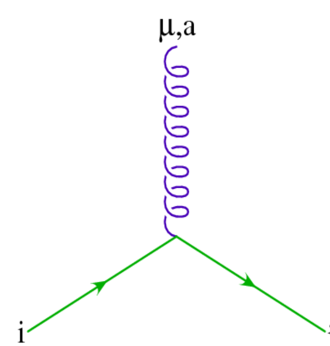
$$\frac{i\delta^{ab}}{k^2+i\varepsilon}$$



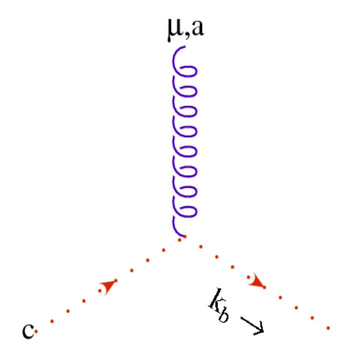
$$g f^{abc}[\eta_{\mu\nu}(k_b-k_a)_\lambda + \eta_{\mu\lambda}(k_a-k_c)_\nu + \eta_{\lambda\nu}(k_c-k_b)_\mu]$$



$$-ig^2[f^{xab}f^{xcd}(\eta_{\mu\lambda}\eta_{\nu\rho}-\eta_{\mu\rho}\eta_{\nu\lambda}) + f^{xac}f^{xbd}(\eta_{\mu\nu}\eta_{\lambda\rho}-\eta_{\mu\rho}\eta_{\nu\lambda}) + f^{xad}f^{xbc}(\eta_{\mu\nu}\eta_{\lambda\rho}-\eta_{\mu\lambda}\eta_{\nu\rho})]$$



$$ig\, T^a_{ji}\gamma_\mu$$



$$g f^{abc} k_{b\mu}$$

Applications of Perturbation Theory to QCD

There are several techniques for applying perturbation theory to QCD:

Fixed Order: All contributions are computed up to a specified order of α_s.

Resummation: For some observables, perturbation theory breaks down due to log enhancements ($\alpha_s \ln \xi \sim 1$). but, one can resum to all orders.

Parton Showers: (See Sjostrand's lectures) Provide more realistic events than fixed order, but are usually based on lowest order matrix elements.

Fixed Order Calculations in QCD

This is the simplest technique and is also the easiest to carry forward to higher orders.

The idea is to compute all quantities up to a certain order of α_s. However, different processes start at different orders of α_s.

Drell-Yan starts at order α_s^0, while n-jet production starts at order α_s^n.

For any process, the lowest non-vanishing order of α_s is called Leading order, or LO.

Higher Order corrections.

For any process, the lowest non-trivial order of α_s is called Leading order, or LO.

Leading Order (LO) calculations are performed at the Born level.

Next-to-Leading Order (NLO) calculations include one-loop corrections to the Born process and Single Real Radiation corrections

Next-to-Next-to-Leading Order (NNLO) calculations include two-loop corrections to Born, one-loop corrections to Single Real Radiation terms and Double Real Radiation correction.

Limitations of Fixed Order Calculations

Experience has shown that LO calculations are of only qualitative value, often getting the normalization and shapes of distributions to within 10-20 percent. Often they do worse.

NLO is the first serious approximation. Unfortunately, the state of the art currently allows for loop calculations with 5 (6 is coming) external partons.

NNLO is only available in a few special cases.

Limitations of Fixed Order Calculations

Many important backgrounds will be computed at NLO. The improved accuracy will be a boon, but fixed order still leaves a lot to be desired in terms of event simulation.

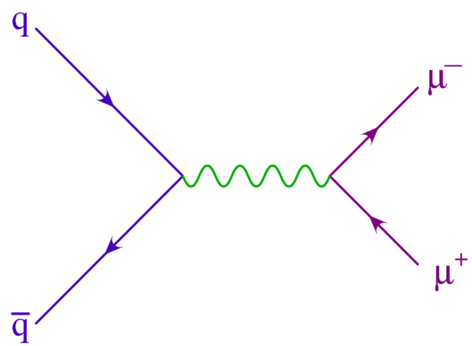
At LO, each parton is identified with a jet. A LOT of structure is being left out. Even at NNLO, a jet can contain at most 3 partons!

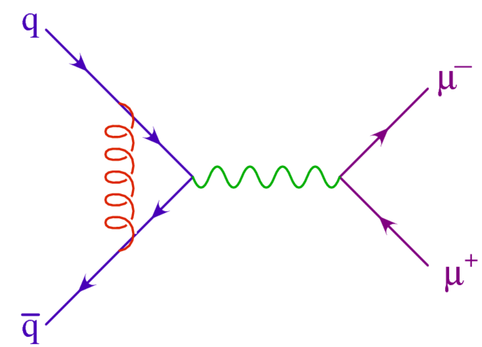
There is great demand for combining the accuracy of NLO with the event simulation of parton showers.

Example: Inclusive Drell-Yan at NLO

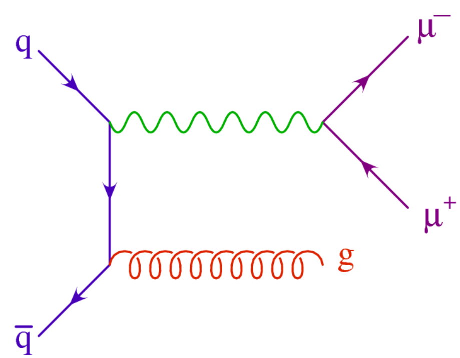
We compute all terms at orders $\alpha_s^{\,0}$ and $\alpha_s^{\,1}$:

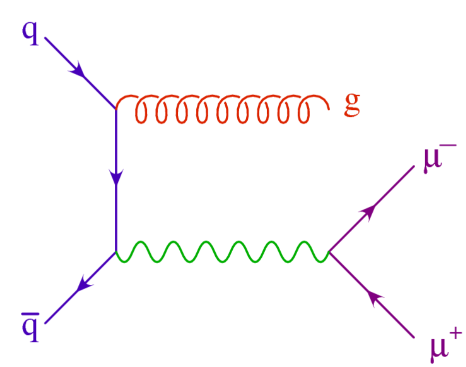
$q\bar{q}$ Born and virtual terms.



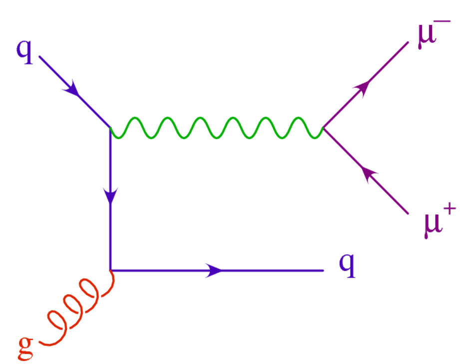


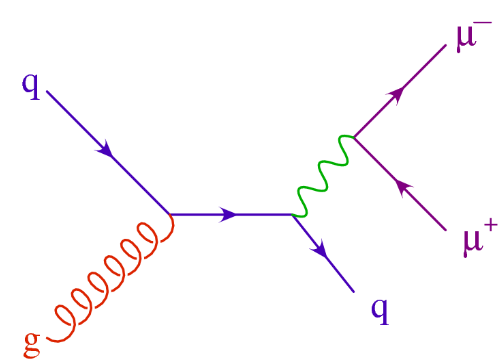
$q\bar{q}$ Real emission terms.





qg Real emission terms.





Drell-Yan at NLO

If we are fully inclusive, we treat the μ pair as a massive vector boson and thus have a 2→1 virtual process. We integrate the squared amplitudes over phase space,

$$\sigma_V = \frac{1}{2\hat{s}} \int \frac{d^{3-2\epsilon} q}{(2\pi)^{3-2\epsilon} 2q^0} (2\pi)^{4-2\epsilon} \delta^{4-2\epsilon}(p_1 + p_2 - q) |M_V|^2$$

$$\sigma_R = \frac{1}{2\hat{s}} \frac{d^{3-2\epsilon} q \, d^{3-2\epsilon} k}{(2\pi)^{3-2\epsilon} 4q^0 k^0} (2\pi)^{4-2\epsilon} \delta^{4-2\epsilon}(p_1 + p_2 - q - k) |M_R|^2$$

and combine real and virtual terms

$$\sigma_{Tot} = \sigma_V + \sigma_{R.}$$

Drell-Yan at NLO

$$\sigma^{(n)} = \sigma_0 \left(\frac{\alpha_s}{\pi}\right)^n \Delta^{(n)}(x), \qquad \sigma_0 = \frac{4\pi\alpha^2}{9Q^4}, \qquad \Delta^{(0)} = \delta(1-x), \qquad x \equiv \frac{Q^2}{\hat{s}}, \qquad \mathcal{D}_n = \left[\frac{\ln^n(1-x)}{1-x}\right]_+$$

$$\Delta^{(1)}_{q\bar{q},V} = C_F\, \delta(1-x)\left[-\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\left(\frac{1}{2} + \ln\frac{\mu^2}{Q^2}\right) - \frac{5}{2} + \frac{7}{2}\zeta_2 - \frac{1}{2}\ln\frac{\mu^2}{Q^2} - \frac{1}{2}\ln^2\frac{\mu^2}{Q^2}\right]$$

$$\Delta^{(1)}_{q\bar{q},R} = C_F\, \delta(1-x)\left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\left(1 - \ln\frac{\mu^2}{Q^2}\right) - \frac{3}{2}\zeta_2 - \ln\frac{\mu^2}{Q^2} + \frac{1}{2}\ln^2\frac{\mu^2}{Q^2}\right] + 2C_F\, \mathcal{D}_0(1-x)\left[-\frac{1}{\epsilon} + 1 - \ln\frac{\mu^2}{Q^2}\right]$$
$$+\ 4C_F\, \mathcal{D}_1(1-x) + C_F(1+x)\left[\frac{1}{\epsilon} - 1 + \ln\frac{\mu^2}{Q^2} - 2\ln(1-x)\right] - C_F\frac{1+x^2}{1-x}\ln(x)$$

$$\Delta^{(1)}_{q\bar{q},V+R} = C_F\, \delta(1-x)\left[-\frac{3}{2}\frac{1}{\epsilon} - \frac{5}{2} + 2\zeta_2 - \frac{3}{2}\ln\frac{\mu^2}{Q^2}\right] + 2C_F\, \mathcal{D}_0(1-x)\left[-\frac{1}{\epsilon} + 1 - \ln\frac{\mu^2}{Q^2}\right]$$
$$+\ 4C_F\, \mathcal{D}_1(1-x) + C_F(1+x)\left[\frac{1}{\epsilon} - 1 + \ln\frac{\mu^2}{Q^2} - 2\ln(1-x)\right] - C_F\frac{1+x^2}{1-x}\ln(x)$$

$$\Delta^{(1)}_{q\bar{q},R} = \frac{T_R}{2}(1 - 2x + 2x^2)\left[-\frac{1}{\epsilon} + 2\ln(1-x) - \ln(x) - \ln\frac{\mu^2}{Q^2}\right] + \frac{T_R}{4}(3 + 2x - 3x^2)$$

The result still has poles in ϵ! Something is missing

Mass Factorization

The parts of the real-emission terms where the final state parton is collinear with the beam has already been included in the parton distributions. Those pieces must therefore be removed from the real emission terms.

This is done by adding in the Mass Factorization Counterterms, which are convolutions of lower-order terms with the DGLAP splitting functions.

$$\sigma_{ij} = \sum_{ij=q,\bar{q},g} \hat{\sigma}_{ab} \otimes \Gamma_{ai} \otimes \Gamma_{bj}, \qquad \Gamma_{ij}(x) = \delta(1-x)\,\delta_{ij} - \frac{\alpha_s}{\pi}\frac{P_{ij}^{(0)}(x)}{\epsilon} + \ldots,$$

$$(f \otimes g)(x) = \int_0^1 dy \int_0^1 dz\, f(y)\, g(z)\, \delta(x - yz)$$

Drell-Yan at NLO

Adding in the Mass Factorization Counterterms

$$\Delta^{(1)}_{q\bar{q},MF} = C_F\left(\frac{1}{\epsilon}-1\right)\left[\frac{3}{2}\delta(1-x)+2\mathcal{D}_0(1-x)-1-x\right]$$

$$\Delta^{(1)}_{qg,MF} = \frac{T_R}{2}\left(\frac{1}{\epsilon}-1\right)\left[1-2\mathrm{x}+2\mathrm{x}^2\right]$$

we get the correct (finite!) result:

$$\Delta^{(1)}_{q\bar{q},Tot} = C_F\left[\left(-4-\frac{3}{2}\ln\frac{\mu^2}{Q^2}+2\zeta_2\right)\delta(1-x)-2\mathcal{D}_0(1-x)\ln\frac{\mu^2}{Q^2}+4\mathcal{D}_1(1-x)\right]$$
$$+\,C_F(1+x)\left[\ln\frac{\mu^2}{Q^2}-2\ln(1-x)\right]-C_F\frac{1+x^2}{1-x}\ln(x)$$

$$\Delta^{(1)}_{qg,Tot} = \frac{T_R}{2}(1-2\mathrm{x}+2\mathrm{x}^2)\left[2\ln(1-x)-\ln(x)-\ln\frac{\mu^2}{Q^2}\right]+\frac{T_R}{4}(1+6\mathrm{x}-7\mathrm{x}^2)$$

Example II: Jet Production

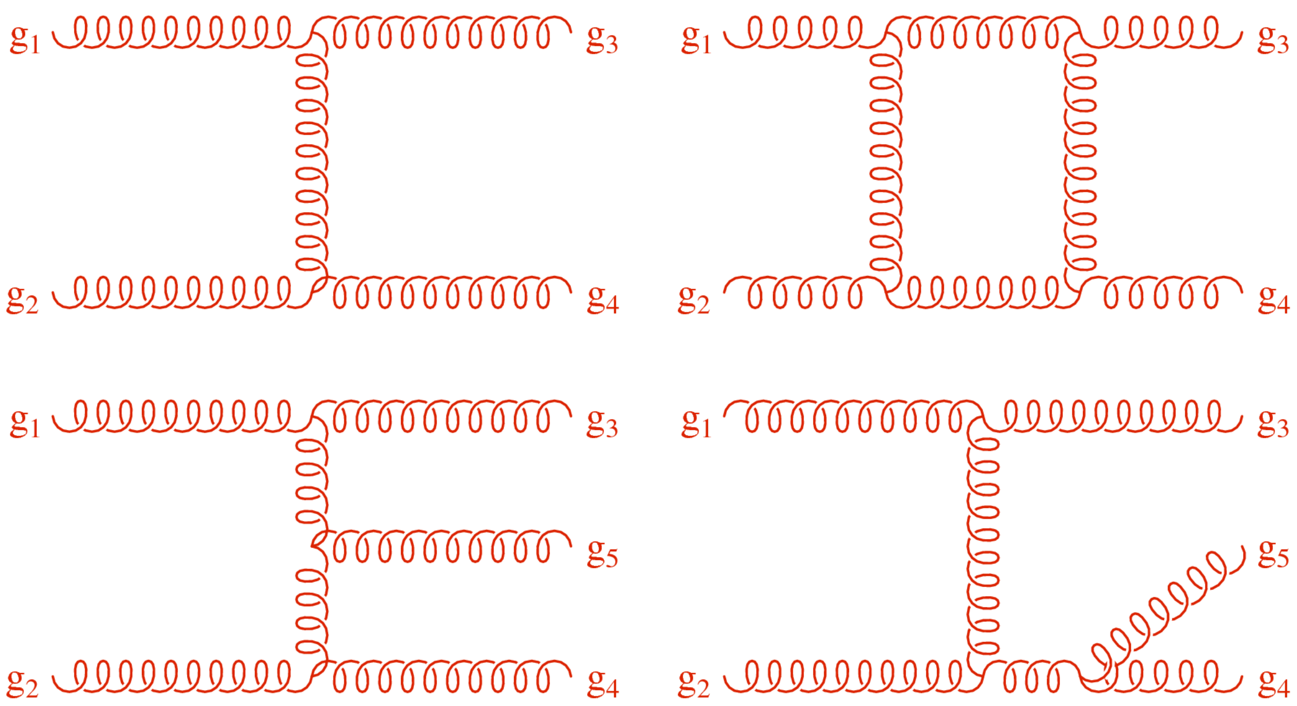
The previous example of an NLO calculation was special for a number of reasons

- There was no need to renormalize
- One could perform the total integrals
- We were not interested in the hadrons in the final state, so we needed no jet algorithms.

Let us now look at jet production.

Jet Production

Again, we have Born, virtual and real emission terms. For simplicity I have drawn only all-gluon diagrams.



Jet Production

When computing jet production, we can't do the total integrals as we did for Drell-Yan.

Even if we could, there is far more information to be had from differential distributions.

To compute differential distributions, we need to impose acceptance cuts, etc., in order to approximate the experimental environment.

This is virtually impossible in an analytic calculation, so we adopt numerical techniques and perform Monte Carlo integrations over phase space.

Numerical Integration at NLO

The problem with numerical integration at NLO is that there are infrared divergences all over the place. The one-loop amplitudes have explicit infrared poles, while the real radiation terms diverge in soft and collinear configurations.

We need some method of regulating the divergences so that we can compute the (finite!) NLO cross section with good numerical accuracy.

Most of all, we would like a flexible algorithm that can be applied to a variety of processes.

Universality of Infrared Structure

It is possible to develop a multipurpose algorithm for NLO calculations because the infrared structure QCD amplitudes is universal and the amplitudes factorize. One loop amplitudes take the form,

$$M^{(1)}(p_1,\ldots,p_n) = V^{(1)}(p_1,\ldots,p_n)\ M^{(0)} + M^{(1),f}(p_1,\ldots,p_n)$$

where V contains all infrared poles and multiplies the Born amplitude. $M^{(1),f}$ is infrared finite.

Universal Infrared Structure

Real radiation amplitudes factorize in the soft and collinear limits.

$$\lim_{p_n \| p_{n+1}} M_{n+1}^{(0)}(p_1,\ldots,p_n,p_{n+1}) = \mathcal{C}(p_n,p_{n+1};K)\times M_n^{(0)}(p_1,\ldots,p_{n-1},K)$$

$$\lim_{p_{n+1}\to 0} M_{n+1}^{(0)}(p_1,\ldots,p_n,p_{n+1}) = \mathcal{S}(p_n,p_{n+1},p_1)\times M_n^{(0)}(p_1,\ldots,p_n)$$

The soft and collinear functions, $\mathcal{S}$ and $\mathcal{C}$, integrated over phase space, generate the infrared poles to cancel those in loop amplitudes.

Integrating over $\mathcal{S}$ and $\mathcal{C}$ to cancel the virtual poles is another way of saying the measurement is "sufficiently inclusive" to be infrared safe.

NLO Jet Production

To summarize: Next-to-Leading Order calculations consist of two contributions:

Virtual Corrections to one loop.

Single Real Emission Corrections at tree-level.

$$\sigma^{NLO} = \int_{n+1} d\sigma_{n+1}^{(0)} + \int_{n} d\sigma_{n}^{(1)}$$

Both terms are infrared singular.

A Multipurpose Approach to NLO

A subtraction scheme adds (and subtracts back out) a local counter-term to both Virtual and Real Correction terms, canceling the infrared singularities.

$$\sigma^{NLO} = \int_{n+1} \left(d\sigma_{n+1}^{(0)} - \quad d\alpha_{n+1}^{(0)} \right) + \int_{n} \; d\sigma_{n}^{(1)} \; + \int_{n+1} d\alpha_{n+1}^{(0)}$$

Both terms are now infrared finite.

The Subtraction Method

Q:How can we construct this local counterterm?

A:The infrared structure of QCD amplitudes is universal.

We define the local counterterm in (n+1) body phase space as

$$A^{(0)}_{n+1}(p_1,\dots,p_{n+1}) = \mathcal{D}(p_n, p_{n+1}, p_1; k_n, k_1)\, M^{(0)}_n(k_1, p_2,\dots,p_{n-1}, k_n) + \dots$$

where $\mathcal{D}$ is a function of p_1, p_n, p_{n+1}, (which define the momenta k_1, k_n) which has the same infrared structure as the real emission amplitude. M_n is the on-shell n-point Born amplitude.

The Subtraction Method

Phase space can be factorized in such a way that the three particle phase sub-space $d\Phi(p_1,p_n,p_{n+1})$ can be integrated down to the two particle subspace $d\Phi(k_1,k_n)$.

Only $|\mathcal{D}|^2$ varies under this integration, which exposes the infrared poles, which cancel those of the loop amplitude as in the KLN theorem.

$$\int_{\Phi_3(p_1,p_n,p_{n+1})/\Phi_2(k_1,k_n)} d\,\Phi_{n+1}(p_1,\ldots,p_{n+1})|A|^2 = V^{(1)}(k_1,\ldots,k_n)|M_n^{(0)}(k_1,\ldots,k_n)|^2$$

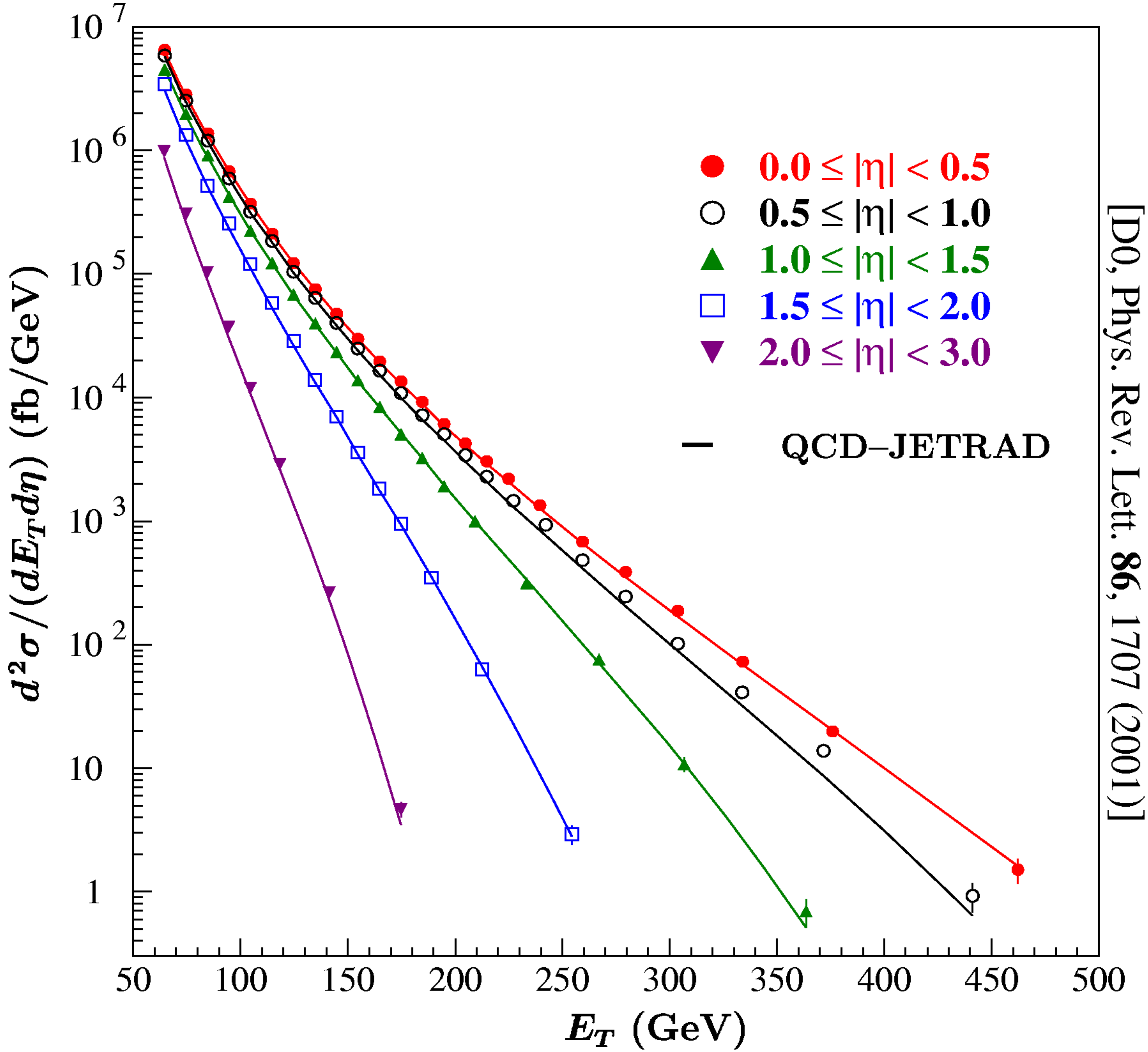
Subtraction at NLO

A subtraction scheme adds (and subtracts back out) a local counter-term to both Virtual and Real Correction terms, canceling the infrared singularities.

$$\sigma^{NLO} = \int_{n+1} \left(d\sigma^{(0)}_{n+1} - d\alpha^{(0)}_{n+1} \right) + \int_{n} d\sigma^{(1)}_{n} + \int_{n+1} d\alpha^{(0)}_{n+1}$$

Both terms are now infrared finite.

Jet Production at the Tevatron



Still to discuss

- Jet algorithms and infrared sensitivity
- Fixed order vs all orders (resummation)

Identified Hadrons

If there are identified hadrons in the final state, (say Υ mesons or photons) these are included through "Fragmentation Functions", which are to some degree the inverse of the parton distributions:

$$D_{h/q_j}(x,\mu) = \frac{1}{12\pi} \sum_X \int dy^- e^{-ip^+ y^-/z} Tr\gamma^+ \langle 0|\psi_j(0,y^-,\boldsymbol{y}_T)\boldsymbol{W}(y^-,0)|h(p)X\rangle$$

$$\times \langle h(p)X|\boldsymbol{W}(y^-,0)\overline{\psi}_j(0)|0\rangle_R$$

The Fragmentation Function $D_{h/c}(z)$ represents the probability of finding hadron h in the decay products (jet) of a parton of type c, carrying fraction z of the parton's momentum